

Flight trajectory data analytics for characterization of air traffic flows: A comparative analysis of terminal area operations between New York, Hong Kong and Sao Paulo[☆]

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ABSTRACT

Future Air Traffic Management systems can benefit from innovative approaches that leverage the increasing availability of operational data to facilitate the development of new performance assessment and decision-support capabilities. This paper presents a data analytics framework for high-fidelity characterization of air traffic flows from large-scale flight tracking data. Machine learning methods are used to exploit spatiotemporal patterns in aircraft movement towards the identification of trajectory patterns and traffic flow patterns. The outcomes and potential impacts of this framework are demonstrated with a comparative analysis of terminal area operations in three representative multi-airport (metroplex) systems of the global air transportation system: New York, Hong Kong and Sao Paulo. As a descriptive tool for systematic analysis of the flow behavior, the framework allows for cross-metroplex comparisons of terminal airspace design, utilization and traffic performance. Novel quantitative metrics are created to summarize metroplex efficiency, capacity and predictability. The results reveal several structural, operational and performance differences between the multi-airport systems analyzed. Our findings show that New York presents the most complex airspace design, with considerably higher number of routes and interactions between them, as well as more dynamic changes in the terminal area flow structure during the day, in part driven by the presence of flow dependencies. Interestingly, it exhibits the best levels of traffic flow efficiency on average, both spatially and temporally, yet the highest variability in metroplex configuration performance, with more pronounced performance degradation during inclement weather.

1. Introduction

A global effort is currently underway towards the modernization and harmonization of Air Traffic Management (ATM) systems in order to enable the foreseen air traffic growth (IATA, 2016) sustainably and address future air transportation challenges. Besides the deployment of new technologies and operational procedures, efficient use and management of air transport system data and information is key to achieve the desired transformation of ATM systems. Every day, large amounts of data are generated as air traffic

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operations occur, both at the planning and execution stages. Data types include environmental conditions (e.g., weather reports), system state (e.g., air traffic control facility reports), flight demand (e.g., airline schedules, flight plans, surveillance tracks), etc. Furthermore, they are expected to become increasingly available and accessible as new technologies are deployed. The development of innovative ways to leverage the raw system data can play a major role in the ATM transformation by providing means for better assessing and understanding operational performance, increasing the system predictive power and creating new decision support tools to assist the planning at strategic and tactical levels.

Flight trajectory data is one example of operational record that has become increasingly accessible with the advent of new surveillance technologies. Historically, there has been limited sharing of this type of data by Air Navigation Service Providers (ANSPs) for national security reasons. With Automatic Dependent Surveillance – Broadcast (ADS-B), open-source flight tracking data has become publicly available, making it possible for the first time to analyze aircraft movement at a global scale. This kind of analysis is of great interest for air traffic management. Unraveling patterns in aircraft movement through advanced trajectory data analytics can contribute to better understand the traffic flow behavior and performance and, in turn, better inform airspace management and traffic flow management related decisions.

This paper presents a data analytics framework for high-fidelity characterization of air traffic flows from large-scale flight tracking data. The framework uniquely applies machine learning methods to extract spatiotemporal patterns in aircraft movement towards the identification of trajectory patterns and traffic flow patterns. The outcomes and potential impacts of this framework are demonstrated with a detailed characterization of terminal area traffic flows in three representative multi-airport (metroplex) systems of the global air transportation system: New York, Hong Kong and Sao Paulo. As a descriptive tool for systematic analysis of the flow behavior, the framework allows for cross-metroplex comparisons of terminal airspace design, utilization and traffic performance. Novel quantitative metrics are created to summarize metroplex efficiency, capacity and predictability. The comparative analysis is used to investigate several structural, operational and performance differences between the metroplexes analyzed.

The terminal area phase is the focus of this work because of its importance in both individual flight and system level efficiency. The high density of aircraft converging in the terminal area along with the more constrained airspace structure often result in sub-optimal flight paths. These inefficiencies are even more pronounced in multi-airport systems (also referred to as metroplexes) serving large metropolitan regions because of the even higher traffic volume and the complex interactions between the arrival and departure flows from the closely located airports. In order to develop new concepts to enhance the efficiency of future metroplex terminal area operations, it is important to develop a deeper understanding of metroplex operational behavior and performance. In this work, we demonstrate how the flight trajectory data analytics framework for characterization of air traffic flows can contribute to this understanding based on a systematic approach backed by operational data.

The remainder of the paper is organized as follows. [Section 2](#) reviews related works in the field of trajectory data mining and provides additional background related to the case studies for this work. [Section 3](#) presents the flight trajectory data analytics framework, describing the methods and datasets used. [Sections 4–6](#) present the results of the characterization of air traffic flows for the multi-airport systems analyzed, describing their structural, operational and performance differences. The results are discussed in [Section 7](#). Finally, [Section 8](#) summarizes the work and suggests future research directions.

2. Background and literature review

2.1. Trajectory data mining

A *flow* can be defined as a pattern in which 1) entities move along the same paths/routes in a spatial dimension and 2) the movements start/end within the same time interval. The identification of air traffic flows from aircraft tracking data can then be framed as a problem of discovery of spatial and temporal trends in the collective movement of aircraft through the airspace. Such problem appears in a variety of other domains in which the movement of objects tends to exhibit correlations in both spatial and temporal dimensions (e.g., vehicles in a road network). Therefore, this section provides a broader review of works (not restricted to the air traffic domain) that aimed to investigate spatial and temporal patterns in trajectory data.

2.1.1. Spatiotemporal pattern recognition in general trajectory data

The problem of discovering spatiotemporal patterns in actual trajectory datasets has received growing attention in the recent literature across a variety of domains. The increased use of position sensing technologies (e.g., Global Positioning System (GPS)) has produced large amounts of data and created an opportunity to gain insights through trajectory data mining. Examples include tracking datasets of vehicles, people, animals, weather phenomena (e.g., hurricanes) etc, which are exploited for a variety of purposes. For instance, analysis of vehicle trajectory data can be used to discover hot spots in a transportation network and support route planning. Analysis of pedestrian flows can help identifying suspicious behavior in a monitored environment; it can also support urban planning and guide infrastructure investments. Analysis of animal movement data can be used to identify regions frequently visited, investigate social structures within a group of animals and better understand migration patterns. Analysis of hurricane/cyclone tracking data can help understanding their movement patterns toward increasing the predictability of severe-weather events for improved disaster relief management.

Early approaches to spatial pattern data mining involved indexing trajectory databases and performing basic analysis such as nearest neighbor queries ([Gudmundsson et al., 2008](#)). More recently, trajectory clustering has been extensively used for the identification of common trajectory patterns and outlier detection in such datasets. [Antonini and Thiran \(2006\)](#) used hierarchical clustering to group trajectories of redundant targets associated with a person for automatic counting of pedestrians in video sequences. [Fu](#)

et al. (2005) also analyzed video surveillance data of real-time traffic to identify vehicle motion patterns. With a first-layer spectral clustering, similar trajectories were grouped together, and with a subsequent hierarchical clustering, dominant paths and lanes were identified. Gaffney and Smyth (1999) and Gaffney et al. (2007) proposed a probabilistic modeling of trajectories as individual sequences of points generated from regression mixture models (the spatial position was modeled with a polynomial regression model in which time was the independent variable). They applied the model-based clustering to identify spatial patterns in extra-tropical cyclone tracks over the North Atlantic using meteorological data. Lee et al. (2007) developed a partition-and-group framework to discover common sub-trajectories as an alternative to clustering trajectories as a whole. They argued that in some applications (especially when there are regions of special interest for the analysis) it may be useful to find portions of trajectories that have similar behavior even if they are dissimilar when compared as a whole. Their proposed algorithm TRACCLUS first partitions a trajectory into a set of line segments at characteristic points where the behavior of the trajectory changes rapidly. Then it groups similar line segments into clusters using a density-based clustering scheme. They used the framework to identify movement patterns in very noisy datasets from hurricane and animal tracking.

Most of the previous research has focused on the identification of similar trajectory patterns in the spatial dimension. A much smaller body of literature has incorporated the temporal dimension in the analysis to investigate trends in the occurrence of spatial clusters over time, which is key for understanding the dynamics of the flow behavior. Kim and Mahmassani (2015) identified spatial travel patterns in a road network with a density-based clustering analysis of vehicle trajectory data and then used k-means to cluster daily time series of traffic for each spatial travel pattern identified towards discovering daily trends in their use. In a similar way, Wen et al. (2016) developed an algorithm to extract shipping route from vessel's historical position point cloud using local polynomial regression and then clustered time series of vessel's frequency of occurrence in multiple zones defined along the frequent routes to find daily traffic patterns.

2.1.2. Spatiotemporal pattern recognition in flight trajectory data

In the aviation domain, clustering techniques have also been extensively used to identify spatial traffic patterns from flight tracking data for performance assessment, airspace monitoring, airspace design and traffic flow management purposes (Eckstein, 2009; Rehm, 2010; Sabhnani et al., 2010; Gariel et al., 2011; Enriquez, 2013; Marzuoli et al., 2014; Murça et al., 2016; Delahaye et al., 2017; Arneson et al., 2017; Bombelli et al., 2017a, 2017b; Andrienko et al., 2018; Ren and Li, 2018). Eckstein (2009) developed a flight track taxonomy for monitoring aircraft behavior based on filtering, segment identification, track decomposition and clustering in order to evaluate how well individual flight trajectories are performing their procedures in the terminal airspace. Gariel et al. (2011) also developed a framework for terminal airspace monitoring using a density-based clustering algorithm to learn typical patterns of operation and to assess the conformance of flight trajectories. Similarly, Rehm (2010) and Enriquez (2013) relied on hierarchical and spectral clustering respectively to identify nominal and abnormal spatial traffic patterns to/from a specific airport. Trajectory clustering has also been used to identify spatial traffic patterns at the en route phase. Sabhnani et al. (2010) developed a greedy grid-based trajectory clustering algorithm in order to learn standard flows and critical points and identify en route sectors with highly structured traffic patterns for the goal of airspace redesign. Delahaye et al. (2017) used hierarchical clustering with a custom distance metric for major flows extraction in the entire French airspace. Arneson et al. (2017) used a density-based clustering algorithm to identify dominant routing structures between the Forth Worth Center and New York Center and assess the impacts of convective weather on the flow rate capacity along the commonly used routes. Similarly, Marzuoli et al. (2014) and Bombelli et al. (2017a, 2017b) clustered trajectories between various origin and destination pairs in order to identify common routing structures and model the air traffic route network towards developing high-fidelity models for traffic flow management optimization. Andrienko et al. (2018) developed a systematic workflow for clustering and visualizing flight trajectories at multiple scales based on user-specified filtering of relevant parts.

Studies that aimed to investigate temporal patterns in the traffic flows are also limited in the aviation domain. Song et al. (2006) manually extracted flow features from actual flight trajectories in a 15-min basis, which were aggregated into a vector-based representation and clustered to identify traffic flow patterns in airspace sectors. Flights entering and exiting an airspace sector through the same neighboring sectors were considered to be part of the same flow regardless the shape of their trajectories. With the methodology, they idealized a framework for sector capacity prediction for strategic time horizon, although little discussion was provided on the predictability of the flow features. Sidiropoulos et al. (2017) proposed a framework to identify spatiotemporal patterns in the traffic crossing the terminal area boundary in metroplex systems using flight plan data. The approach first identified temporal changes in the local demand detected for discretized zones of the terminal area boundary using a threshold for the difference in the number of flights between consecutive time periods. Subsequently, temporal clusters in the aggregate metroplex demand were identified using a threshold on the number of zones with detected local demand change. These thresholds were optimized to account for uncertainty in demand data. Finally, within each temporal cluster, flights were spatially clustered based on the direction in which they enter or exit the terminal area boundary using k-means algorithm. With the framework, their goal was to provide enhanced demand estimates over shared arrival and departure fixes for supporting metroplex traffic flow management. However, as the spatial analysis was restricted to the location of crossings at the terminal area boundary, their framework did not provide complete flow information.

The flight trajectory data analytics framework presented in this paper uniquely incorporates both spatial and temporal dimensions in the investigation of aircraft movement patterns towards enabling a high-fidelity characterization of air traffic flows.

2.2. Terminal area operations in multi-airport systems

The proposed flight trajectory data analytics framework is demonstrated at the terminal area scale, particularly of multi-airport systems, in this study. A multi-airport system (also referred to as metroplex) can be defined as a set of two or more significant airports that serve commercial traffic in a metropolitan region, without regard to ownership or political control of individual airports (DeNeufville and Odoni, 2013). Multi-airport systems have emerged worldwide as a response to congestion problems, allowing the air transportation system to scale and meet the increasing demand (Bonnefoy, 2008). With the emergence of multi-airport systems, the ATM system has had to deal with increasing levels of complexity in the management of the traffic surrounding these systems. The sharing of the terminal airspace results in operational interdependencies and interactions between dense arrival and departure routes from the multiple airports in the same geographical area, creating a complex environment for traffic management.

The metroplex airspace design and the operating strategies governing the use of the airspace are recognized to be key factors affecting the coupling of operations, the traffic dynamics and the overall performance of a metroplex. Yet, studies that investigate the interplay between these factors and their impacts on the traffic dynamics and performance are limited. Visual inspection of flight trajectory data has been used to qualitatively evaluate airspace design and identify potentially constraining flow interactions (Ren et al., 2009). An extensive study developed by Clarke et al. (2012) identified major metroplex dependency issues that impact their operations based on site visits, domain expert evaluation and qualitative analysis of four major U.S. multi-airport systems (Atlanta, Los Angeles, New York and Miami). Delay and fuel burn impacts of decoupled metroplex design and new traffic scheduling strategies have been assessed through simulation (Clarke et al., 2012). Donaldson and Hansman (2010) presented an empirical study aimed at quantifying capacity impacts associated with metroplex interactions. Throughput performance at the airport level and at the system level for particular combinations of runway configurations in the New York metroplex were evaluated using operational data. Capacity discrepancies as great as 60 operations per hour were observed, emphasizing that terminal airspace flow interactions tend to be an important constraining factor driving the capacity of metroplex airports.

In this work, we demonstrate how the flight trajectory data analytics framework for characterization of air traffic flows can be used to obtain a deeper understanding of metroplex operational behavior and performance based on a systematic approach backed by operational data.

2.3. Case studies

Three representative multi-airport systems of the global air transportation system were selected for a comparative analysis of traffic flows: New York, Hong Kong and Sao Paulo. Fig. 1 shows the lateral trajectories of arrival and departure flights during one day of terminal area operations in these regions and provides a good illustration of the air traffic density and complexity that characterize metroplex systems.

The multi-airport system that serves the New York metropolitan region is composed of three primary commercial airports: John F. Kennedy International (JFK), Newark International (EWR) and LaGuardia (LGA). Together, they served 128.9 million passengers in 2016, being the world's second busiest multi-airport system, after London (ACI, 2016). Given the high volume of operations (which is accentuated by the presence of other smaller airports serving the demand for business and charter flights such as Teterboro airport) and the proximity between the airports (LGA is located 10 miles from JFK and 16 miles from EWR), the New York metroplex is often recognized as one of the most operationally complex multi-airport systems in the world.

The Hong Kong metropolitan region is served by two primary commercial airports, Hong Kong International (HKG) and Shenzhen Bao'an International (SZX), and a secondary airport, Macau International (MFM). As a major gateway to China and Asia, HKG is one of the top-ten international airports in terms of passenger movements and it is the world's largest air cargo hub (Hong Kong

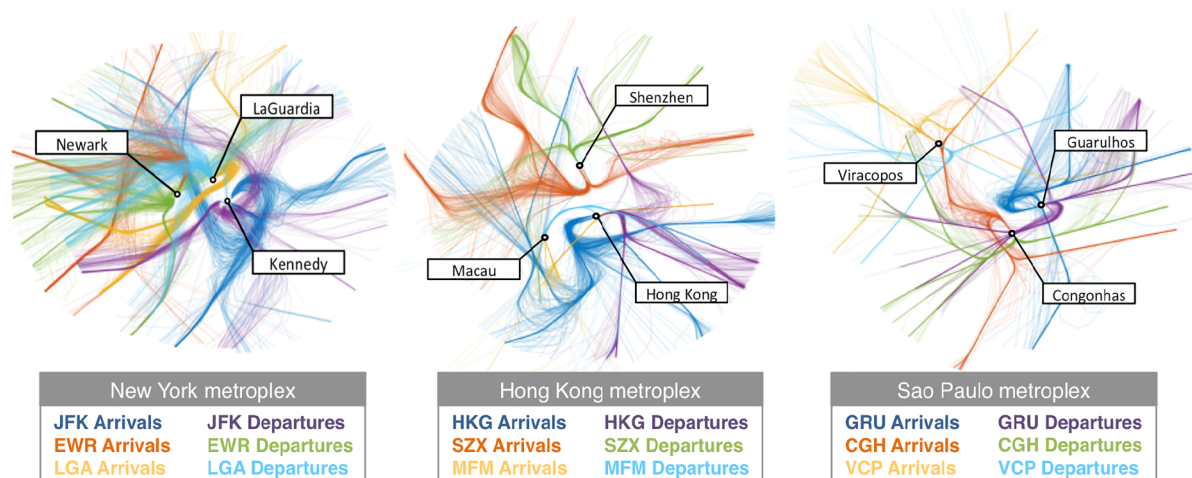


Fig. 1. Arrival and departure trajectories for one day of terminal area operations in the New York, Hong Kong and Sao Paulo metroplexes.

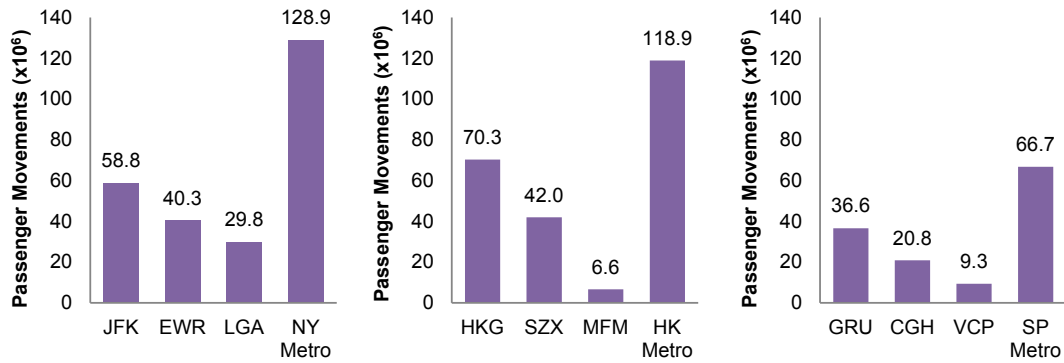


Fig. 2. Passenger movement by airport at the New York, Hong Kong and Sao Paulo metroplexes in 2016.

International Airport, 2016). Together with SZX and MFM, the Hong Kong multi-airport system served more than 100 million passengers in 2016 and it is one of the busiest systems in the world.

Finally, the multi-airport system that serves the Sao Paulo metropolitan region is composed of two primary commercial airports, Sao Paulo/Guarulhos International (GRU) and Sao Paulo/Congonhas (CGH), and a secondary airport, Viracopos International (VCP). Although VCP is located 50 miles from the Sao Paulo city center, today it serves an important part of the metropolitan region air travel demand given the physical limitations for capacity expansion at the primary airports. The Sao Paulo metroplex is the busiest multi-airport system in Latin America, with GRU and CGH being the busiest airports in Brazil in terms of passenger movement.

The detailed passenger and aircraft movement statistics for the year 2016 (ACI, 2016; PANYNJ, 2016; Hong Kong International Airport, 2016; Civil Aviation Resource Network, 2016; Macau International Airport, 2016; GRU Airport, 2016; Aeroportos Brasil Viracopos, 2016; Infraero Aeroportos, 2016) at these three multi-airport systems is presented in Figs. 2 and 3. The New York system is the busiest, followed by Hong Kong and Sao Paulo. The Hong Kong system has the highest ratio between passenger movements and aircraft movements, indicating higher participation of larger aircraft in its mix of operations.

3. Methodology

3.1. Data description

The data analytics framework aims at providing a high-fidelity traffic flow characterization by exploiting flight tracking data. This type of data is continuously generated by surveillance systems (e.g., radar, ADS-B) responsible for tracking aircraft through the airspace and enabling real-time monitoring and control of the traffic around the world. In order to apply the framework for the multi-airport systems analyzed in this paper, the following sets of data were used: for the New York metroplex, a set of 69 days of flight tracks from the Traffic Flow Management System (TFMS), made available by the Federal Aviation Administration (FAA); for both the Hong Kong and Sao Paulo metroplexes, a set of 60 days of flight tracks from the *FlightRadar24* global flight tracking service (FlightRadar24, 2018). The datasets report one-minute updates of aircraft state, including flight ID, latitude, longitude, altitude, speed, origin airport, destination airport and aircraft type. Dates were empirically selected to ensure coverage of a broad set of operational conditions and enable the characterization of the traffic flows and their performance for days with and without weather-related constraints; 55% are fair weather condition days and 45% are weather-impacted days (further categorized as convective weather impacts - 30% - and non-convective weather impacts, such as adverse wind conditions or low ceiling/visibility - 15%). Historical Meteorological Terminal Aviation Routine Weather Report (METAR) data was used to categorize the weather conditions

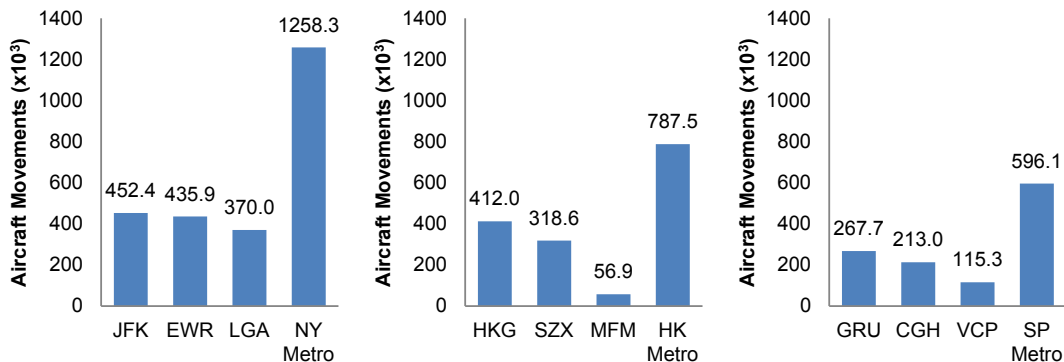


Fig. 3. Aircraft movement by airport at the New York, Hong Kong and Sao Paulo metroplexes in 2016.

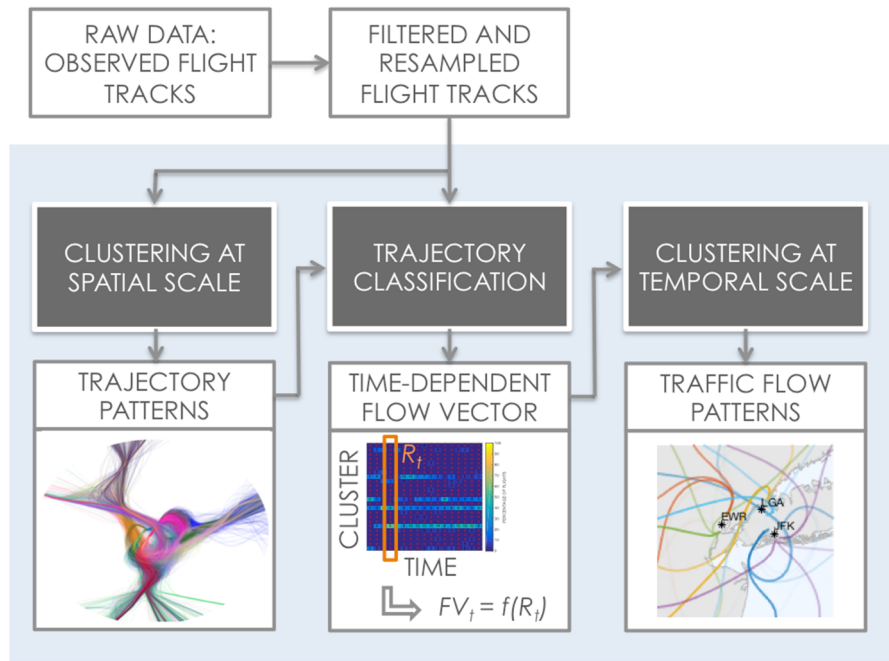


Fig. 4. Schematic overview of the flight trajectory data analytics framework for characterization of air traffic flows.

for each day of operations.

Sanity checks were performed to ensure that the data quality was appropriate for the purpose of this study. First, the coverage of *FlightRadar24* data was checked by comparing the flights tracked by the *FlightRadar24* network with the actual departure and arrival information from the airports' websites. The analysis was performed for the Hong Kong metroplex. The percentage of flights captured by the *FlightRadar24* data was 97%, 89%, and 93% for HKG, SZX and MFM, respectively. Second, both the TFMS data and the *FlightRadar24* data are subject to error, noise, and gaps in aircraft position measurements. The Despeckle filter proposed by Palacios and Hansman (2013) was used to correct altitude errors that were particularly present in the TFMS data. Remaining few data gap irregularities were smoothed during the trajectory resampling process described in Section 3.2.1.

3.2. Flight trajectory data analytics framework for characterization of air traffic flows

The data analytics framework for characterization of air traffic flows is composed of three modules that use machine learning techniques to mine spatial and temporal patterns in flight tracking data. It is illustrated in Fig. 4. In the first module, clustering at spatial scale is performed with a trajectory clustering scheme to identify spatial patterns of aircraft movement, which are referred to as *trajectory patterns* and define the as-flown route structure in the airspace of interest. Based on this knowledge, the second module uses a trajectory classification scheme to match new flight trajectories with the learned airspace structure and classify them as conforming (to one of the learned routes) or non-conforming. With trajectories classified, flows are identified as temporally associated flight trajectories conforming to the same standard route. The last module of the framework seeks to discover patterns in the traffic flow behavior over time. For this, a clustering analysis at temporal scale is performed to identify similar structures in the set of flows observed during a period of time, which are referred to as *traffic flow patterns*. A detailed description of each module of the framework is provided next.

3.2.1. Clustering at spatial scale: trajectory clustering

In the first module of the framework, a trajectory clustering scheme is developed to identify spatial patterns of aircraft movement. Clustering is an unsupervised learning method that aims to identify groups of similar observations in a dataset without prior knowledge about the existence of these groups or about how the observations are distributed among them. In the trajectory clustering problem, the goal is to find groups of similar trajectories in the spatial dimension. We define a group of spatially similar trajectories as a *trajectory pattern*.

Following the typical clustering methodology, the trajectory clustering process requires a data representation, a similarity/distance function and a clustering method. The raw flight trajectory data are time-series of aircraft position sampled every minute. First, a filtering procedure is implemented to extract the portion of the trajectory associated with the airspace region of interest. In our application, for extracting the terminal area phase, we considered the trajectory information between the airport runway threshold and the terminal area boundary, which was modeled as a circle of 60-mile radius with its center at the airport. The filtered flight trajectories are characterized by time-series of different lengths, depending on the time spent in the airspace volume under

consideration. Data resampling is then performed to transform each time-series into a high-dimensional feature vector of fixed dimension and enable the assessment of similarity between flight trajectories using standard Euclidean distance. The resampling approach normalizes the time stamps for each trajectory into the interval $\tau = [0,1]$, divides τ into a fixed number of equally sized time blocks and linearly interpolates the spatial position for the fixed number of normalized time stamps in τ . The result is a feature vector of 2D spatial position evenly spaced in time $F_i = (x_{i1}, y_{i1}, x_{i2}, y_{i2}, \dots, x_{in}, y_{in})$.

The final requirement in the clustering methodology is the method for grouping similar observations (Jain et al., 1999; Jain, 2010). A density-based clustering algorithm – Density-Based Spatial Clustering of Applications with Noise (DBSCAN) (Ester et al., 1996) – is used for flight trajectory clustering. As the name of the algorithm suggests, this method is suitable for datasets with noise. In flight trajectory datasets, the standard routes and adaptations produce the core underlying patterns, yet abnormal trajectories can also occur for a variety of reasons and can be considered as noise. DBSCAN enables the identification of the core trajectory patterns in the presence of abnormal trajectory profiles. Other advantages of DBSCAN include the ability to discover non-convex clusters and no need to set the number of clusters a priori.

DBSCAN relies on two input parameters in order to cluster the data space:

- *MinPts*: a minimum number of points (observations);
- ϵ : a distance threshold.

Using these parameters, the algorithm is built on the three following fundamental concepts:

(1) ϵ -neighborhood

The ϵ -neighborhood of an observation F_i in a dataset D contains all the neighboring observations that are within a distance ϵ and is defined in (1).

$$N_\epsilon(F_i) = \{F_j \in D / d(F_i, F_j) \leq \epsilon, d(F_i, F_j) = \|F_i - F_j\|_2\} \quad (1)$$

(2) Density-reachability

An observation F_j is directly density-reachable from an observation F_i if:

- $F_j \in N_\epsilon(F_i)$
- $|N_\epsilon(F_i)| \geq \text{MinPts}$ (core point condition)

An observation F_j is density-reachable from an observation F_i if there exists a chain F_i, \dots, F_j such that each observation is directly density-reachable from the predecessor.

(3) Density-connectivity

An observation F_j is density-connected to an observation F_i if there exists another observation F_k such that both F_j and F_i are density-reachable from F_k . A cluster is then defined as a set of density-connected observations.

Based on these concepts, the algorithmic clustering process starts by visiting an arbitrary instance of the database and determining its ϵ -neighborhood. If it contains at least *MinPts* (core point condition), the observation and its neighbors start a cluster. The ϵ -neighborhood of the neighbors is iteratively retrieved and the same procedure is applied until all the border points of the cluster are achieved. Otherwise, the observation is labeled noise and the next instance of the database is visited. It is worth mentioning that an instance labeled noise can later become part of a cluster if it is reachable from a new discovered core point.

The clustering solution provided by DBSCAN is dependent on the input parameters *MinPts* and ϵ . DBSCAN is not particularly sensitive to the *MinPts* choice, which will determine the smallest number of observations to define a cluster. Once *MinPts* is chosen, the choice of ϵ will involve a trade-off between undesirable creation of clusters from minor variations in a major cluster and undesirable merger of distinct clusters. In our application of the framework for the characterization of metroplex flows, quantitative evaluation of clustering quality using validity indices – Silhouette plots (Rousseeuw, 1987) – and visual inspection were performed on a case-by-case basis in order to tune the parameters of the algorithm.

3.2.2. Trajectory classification

In the second module, a trajectory classification scheme is developed to match new flight trajectories with the airspace structure identified in the first module, including the ability to identify non-conforming behaviors (those that do not conform with one of the identified trajectory patterns). The purpose of the trajectory classification module is two-fold: (1) given the computational effort associated with the trajectory clustering process, the classification procedure provides a more efficient way for processing large sets of trajectory data for spatial pattern identification during offline applications; (2) although outside the scope of this paper, it is key for online applications that depend on real-time flow identification (e.g., airspace monitoring), as it enables consistent processing of new batches of trajectory data continuously generated by the surveillance system.

Classification is a supervised learning method that aims at predicting the class/label of an observation given a set of predictors/

features based on prior knowledge extracted from a training dataset. In the trajectory classification problem, the classes are defined by the trajectory patterns learned with the spatial clustering analysis (training dataset). Given a new flight trajectory defined by its vector of spatial position, the goal is to predict whether it conforms to one of the learned trajectory patterns. Obviously, not all the flight trajectories will match the airspace structure identified and the classification scheme should also have the ability to detect these non-conforming behaviors.

The trajectory classification process starts with the development of a classifier that can successfully discriminate the learned trajectory patterns. A multi-way classifier using the Random Forests algorithm is created. Random Forests is selected because it is widely recognized as one of the highest performing methods in terms of classification accuracy and because it runs efficiently on large databases. Random Forests is a non-parametric method for both classification and regression consisting of an ensemble of decision tree learners (Breiman, 2001). A decision tree is defined by a hierarchical structure of decision nodes that progressively partitions the input space using a sequence of binary splits from the root node to the leaves (terminal nodes). Each decision node evaluates the value of a feature from the input set to decide which branch will be taken from it. The goodness of a split is quantified with an impurity measure. In a classification setting, the Gini index is typically used. At each decision node, the method searches for the partition that will minimize the impurity among all input variables and their potential splits. The splitting process is repeated until a stopping criterion is met (e.g., node is pure, minimum number of observations is achieved). The result is a rectangular partition of the input space. Every leaf node l then corresponds to a rectangular subspace R_l . For every training observation x_i , there is only one leaf node such that $x_i \in R_l$. The prediction for a new observation x' is obtained by applying a majority rule for the leaf $l(x')$ that this observation falls into when it is passed through the tree. As any ensemble method, Random Forests creates multiple decision trees with bootstrap samples of the training data, but it has the unique characteristic of analyzing only a random subset of the features to determine each split. The final response of the model is computed by aggregating the results of the individual trees using a majority rule (higher percentage of tree votes in the ensemble). The algorithm has few parameters to tune: T is the number of trees in the ensemble, M is the number of features to sample at each split and N is the minimum number of observations in a leaf node. In our application, the Random Forests classifiers were created with $T = 50$, $M = 10$ and $N = 1$, based on 5-fold cross-validation performance.

The Random Forests classifier is used to discriminate the spatial patterns between the conforming trajectories. However, given a new flight trajectory, it is not known a priori whether it conforms to the standard routes. In order to enable the identification of non-conforming behaviors, the second module also includes a semi-supervised anomaly detection framework based on Conformal Prediction. Conformal Prediction is a technique that intends to generate confidence intervals for predictions made by machine learning algorithms (Shaffer and Vovk, 2008). Therefore, it is built upon existing classification and regression methods. In a classification setting, the concept involves the estimation of a p-value for each candidate label that can be attributed to a test example based on a non-conformity metric, which measures how different this observation is relative to a set of training examples that have the candidate label. The labels with a p-value lower than the specified significance level ε are then excluded from the prediction set.

Formally, given a training dataset $Z: (x_1, y_1), \dots, (x_{n-1}, y_{n-1})$, where $x_i \in X$ is a vector of attributes and $y_i \in Y$ is the class for observation i , non-conformity scores α_i can be calculated for each observation based on non-conformity functions. These non-conformity functions are typically tailored to the machine learning algorithm used to create the mapping between the input space and the output. Since Random Forests is the underlying classification method for conformal prediction, a non-conformity score based on the proportion of tree votes in the ensemble is used (Devetyarov and Nourtdinov, 2010; Bhattacharyya, 2013; Johansson et al., 2014):

$$\alpha_i = 1 - (|\text{tree } j \in \text{ensemble}: y_i = \hat{y}_i^j| / |\text{ensemble}|) \quad (2)$$

According to Eq. (2), the lower the agreement between the trees with respect to the label of a particular example, the higher the non-conformity score for this example. For a new observation $z_n = (x_n, \hat{y}_n)$ for which we want to predict the label, a non-conformity score $\alpha_n^{\hat{y}_n}$ can be calculated for each tentative label \hat{y}_n and compared with the non-conformity scores of the training dataset in order to generate an associated p-value:

$$p(\hat{y}_n) = (|\{z_i \in Z: \alpha_i \geq \alpha_n^{\hat{y}_n}\}| + 1) / (|Z| + 1) \quad (3)$$

The p-value is a measure of confidence in the prediction. The higher the p-value, the higher the confidence. For a significance level ε , if $p \geq \varepsilon$, \hat{y}_n becomes part of the prediction set with confidence $1 - \varepsilon$. Non-conforming flight trajectories can be identified when the p-value is lower than the significance level for all possible labels, i.e., when the prediction set is empty. In contrast to previous methods, a key property of conformal anomaly detection is that it provides a well-founded approach for tuning the confidence thresholds for anomaly detection that can be directly related to the expected or desired false alarm rate.

Once trajectories are classified, flows can be identified as temporally associated flight trajectories conforming to the same standard route. For this, the output of the trajectory classification is organized and stored in daily flow matrices $W \in {}^{n \times p}$, with n rows (number of trajectory patterns) and p columns (number of time periods during the day). Each matrix element w_{ij} indicates the number of trajectories conforming to trajectory pattern i during time period j . A flow is identified whenever $w_{ij} > 1$. The matrix columns are vectors $\{R_t \in {}^n, t = 1, \dots, p\}$ that indicate the time-dependent traffic flows.

3.2.3. Clustering at temporal scale: time-dependent flow vector clustering

The third module in the trajectory data analytics framework has the goal of identifying patterns in the traffic flow structure (group of traffic flows in a given time period). For this, a second-layer clustering analysis is performed on the set of time-dependent traffic

flows R_t .

In order to enable the clustering of the metroplex traffic flows in our application, a data representation was established for the time-dependent metroplex flow structure. The columns $\{(R_k^{m,Arr})_t \in |K^{m,Arr}|, t = 1, \dots, p\}$ and $\{(R_k^{m,Dep})_t \in |K^{m,Dep}|, t = 1, \dots, p\}$ of the daily arrival and departure flow matrices created for each metroplex airport m (where $K^{m,Arr}$ and $K^{m,Dep}$ are the sets of arrival and departure trajectory patterns for airport m , respectively) were aggregated into flow vectors to represent the hourly terminal area flow structure in the metroplex:

$$\{FV_t = ((C_i^{m,Arr})_t, (C_j^{m,Dep})_t), m \in \mathcal{M}, i \in A, j \in D, t = 1, \dots, p\}$$

with:

$$(C_i^{m,Arr})_t = \begin{cases} \operatorname{argmax}_{k \in K_i^{m,Arr}} \{(R_k^{m,Arr})_t\}, & \text{if } \max_{k \in K_i^{m,Arr}} \{(R_k^{m,Arr})_t\} > 1 \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

$$(C_j^{m,Dep})_t = \begin{cases} \operatorname{argmax}_{k \in K_j^{m,Dep}} \{(R_k^{m,Dep})_t\}, & \text{if } \max_{k \in K_j^{m,Dep}} \{(R_k^{m,Dep})_t\} > 1 \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

where \mathcal{M} is the set of metroplex airports, A is the set of arrival gates in the terminal area boundary, D is the set of departure gates, $K_i^{m,Arr} \subset K^{m,Arr}$ is the set of arrival trajectory patterns between arrival gate $i \in A$ and airport $m \in \mathcal{M}$, and $K_j^{m,Dep} \subset K^{m,Dep}$ is the set of departure trajectory patterns between airport $m \in \mathcal{M}$ and departure gate $j \in D$. Arrival/departure gates are defined as zones in the terminal area boundary intersected by a group of trajectory patterns.

It follows that the hourly flow vector FV_t is a categorical vector of dimension $|\mathcal{M}| |A| + |\mathcal{M}| |D|$ and indicates the dominant arrival flows from each arrival gate to each airport and the dominant departure flows from each airport to each departure gate.

Hierarchical clustering was applied using the Hamming distance and complete linkage for clustering the set of hourly flow vectors. This clustering algorithm creates a hierarchical decomposition of the data space (usually represented by a dendrogram) based on a dissimilarity matrix that progressively splits the database into smaller subsets using either a divisive (from the root to the leaves) or an agglomerative (from the leaves to the nodes) approach (Jain et al., 1999). Under the complete linkage agglomerative approach, distances between groups of observations are calculated as the maximum distance between elements in these groups. A distance threshold is then used to horizontally cut the dendrogram and determine the clusters. The Hamming distance was used to assess the similarity between the categorical flow vectors and it corresponds to the number of elements in which they differ. In this case, zero entries indicate absence of flow between the arrival/departure gate and the airport and were disregarded during the computation of the Hamming distance.

4. Characterization of structural differences through the analysis of terminal area route structures

4.1. Identification of trajectory patterns

The first module of the framework was used to identify spatial trajectory patterns in the terminal area. A subset of the data was used to reduce the computational effort. For the New York metroplex airports, a set of sixteen days was used. For the Hong Kong and Sao Paulo metroplex airports, given their lower number of operations, a set of thirty days was used. In order to increase the spatial clustering quality, the trajectory clustering analysis was performed for each airport and for each type of operation (arrival or departure) separately. This section is focused on the results related to arrival trajectories, but a similar analysis was conducted for departures. Table 1 shows the DBSCAN parameter setting used in the clustering process of arrival trajectories at each airport and the resulting clustering output in terms of number of clusters identified and percentage of noise (unstructured data). As an example, Figs. 5–7 show the arrival clusters identified for the major airport in each metroplex. The distribution of observations in the clusters is also presented. For instance, it is observed that both HKG and GRU have one dominant trajectory pattern that concentrates more than 30% of the observations, while JFK has a lower concentration of trajectories in one particular pattern. It is also observed that JFK has

Table 1
DBSCAN parameter settings used for trajectory clustering and corresponding output for each metroplex airport.

Airport	Number of observations	MinPts	Epsilon	Number of clusters	Percentage of noise
JFK	6917	6	1.4	20	14.0
EWR	7136	6	1.4	14	13.8
LGA	6709	6	1.2	16	13.5
HKG	11,023	6	1.4	15	20.8
SZX	8686	6	1.4	8	10.4
MFM	1718	6	1.6	6	29.1
GRU	7816	6	1.4	10	22.9
CGH	5082	6	1.4	6	12.9
VCP	2239	6	1.4	8	16.1

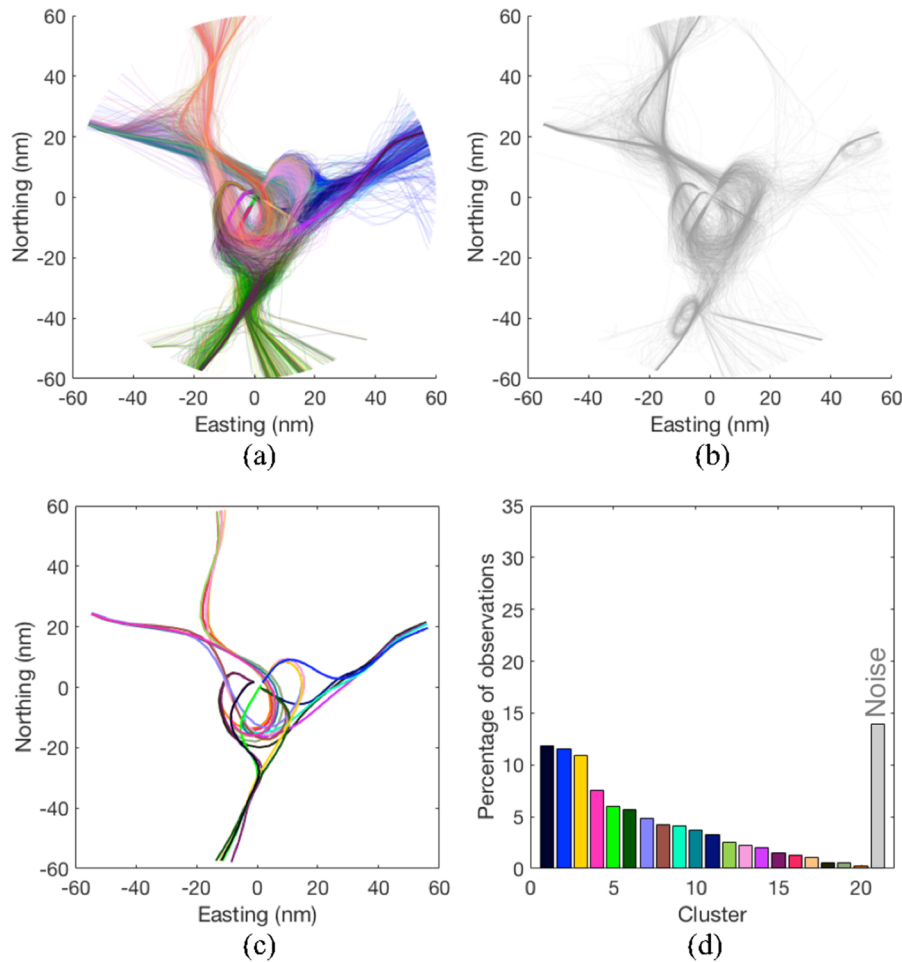


Fig. 5. Results of the trajectory clustering analysis for JFK arrivals. (a) Trajectory clusters; each color represents one cluster. (b) Trajectories labeled as noise. (c) Trajectory cluster centroids. (d) Distribution of trajectories by cluster.

a lower percentage of noise observations than HKG and GRU.

A total of 50, 29 and 24 arrival trajectory clusters was identified for the New York, Hong Kong and Sao Paulo multi-airport systems, respectively. The centroids of these clusters are shown in Fig. 8. At a first glance, it appears that the New York metroplex has the most complex airspace structure. First, there are more trajectory patterns, which can be explained not only because of the higher number of runways, but also because of the presence of more than one trajectory pattern for some combinations of arrival/departure gate and runway threshold. An example is shown in Fig. 9 for LGA arrival trajectories. Two trajectory patterns were identified for flights entering the terminal area through the west gate and landing at runway 22, which are driven by different maneuvers to intercept the final approach leg. Second, the trajectory patterns are more closely located, with a higher number of lateral crossings.

4.2. Identification of route intersections

The observed lateral crossings of trajectory patterns in Fig. 8 do not necessarily indicate they are closely located at the vertical dimension to potentially create flow interactions. In order to investigate the complexity of the airspace structure in terms of the potential to create flow interactions between the airports, we introduce the concept of trajectory tubes. A trajectory tube is defined for each spatial cluster based on the dispersion of their trajectory members. Since all trajectories are described with the same number of resampling points $i = 1, \dots, n$, the tubes are discretized in $(n-1)$ parts. Each discrete part $i = 1, \dots, (n-1)$ is determined by resampling points i and $i+1$. For each part i , the width of the tube is determined by the 95th percentile of the lateral spread of all i^{th} and $(i+1)^{\text{th}}$ resampled points around the cluster centroid. The height of the tube is determined by the difference between the 95th and the 5th percentile of all i^{th} and $(i+1)^{\text{th}}$ vertical positions.

We calculated the intersection volume between all pairs of trajectory tubes from different airports. They indicate the terminal airspace regions where the metroplex trajectory patterns intersect laterally and vertically and flows can potentially interact. It should be mentioned that the temporal dimension is not considered at this point of the analysis, but is incorporated later with the analysis of metroplex traffic flow dynamics. The reader is referred to Section 5.2, which investigates the presence of flow interactions associated

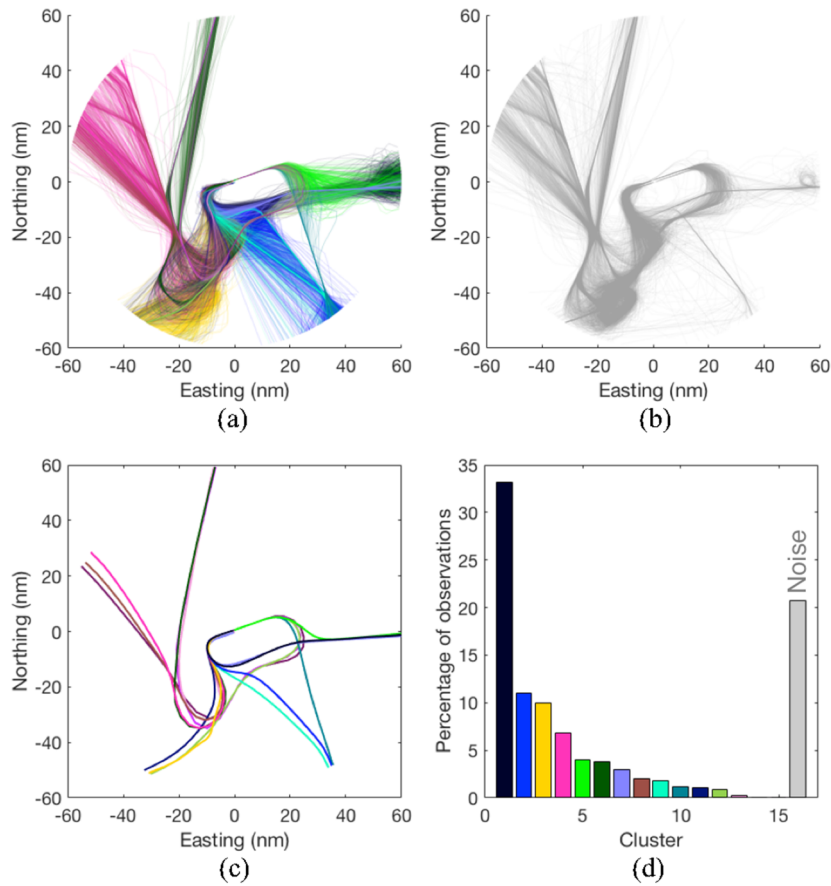


Fig. 6. Results of the trajectory clustering analysis for HKG arrivals. (a) Trajectory clusters; each color represents one cluster. (b) Trajectories labeled as noise. (c) Trajectory cluster centroids. (d) Distribution of trajectories by cluster.

with the trajectory tube intersections. Table 2 shows the number of trajectory tube intersections identified by pair of metroplex airports and Fig. 10 shows their location and size. First, it is noticeable that the New York metroplex has the most conflicted airspace structure. JFK and LGA are the pair of airports with the highest number of intersections. A significant number of the New York metroplex route intersections are located less than 10 nautical miles from the airports (especially for JFK and LGA). An example is shown in Fig. 11. It displays the intersection of a LGA arrival trajectory tube and a JFK arrival trajectory tube with the airspace volume colored in red. It is also observed some clusters of high volume trajectory tube intersections located close to the terminal area boundary, revealing the sharing of arrival/departure gates.

The Hong Kong metroplex shows the least conflicted airspace structure. The airspace structures of the two primary airports are highly de-conflicted, with the exception of a small cluster of intersections between HKG arrival trajectory patterns and SZX departure trajectory patterns located in the south departure area of SZX. Most of the intersections are located more than 10 nautical miles away from the airports. A few high volume intersections are observed between 10 and 30 nautical miles from the airports, revealing the sharing of route segments. An example of a shared route segment between a SZX arrival trajectory tube and a MFM arrival trajectory tube is shown in Fig. 11. It is also observed some clusters of high volume intersections located close to the terminal area boundary, revealing the sharing of arrival/departure gates, especially between SZX and MFM.

Finally, in the Sao Paulo metroplex, GRU and CGH have the highest number of intersections. Most of these intersections are located close to the terminal area boundary, revealing the sharing of arrival/departure gates. Fig. 11 shows an example of departure trajectory patterns from GRU and CGH sharing the same departure gate. A few high volume intersections are also observed between 10 and 30 nautical miles from the airports, revealing the sharing of route segments.

5. Characterization of operational differences through the analysis of traffic flow dynamics

5.1. Identification of metroplex flow patterns

Once the metroplex airspace structures were identified, Random Forests classifiers were created to discriminate the trajectory patterns for each type of operation (arrival/departure) at each metroplex. All trajectory classifiers showed multi-way classification

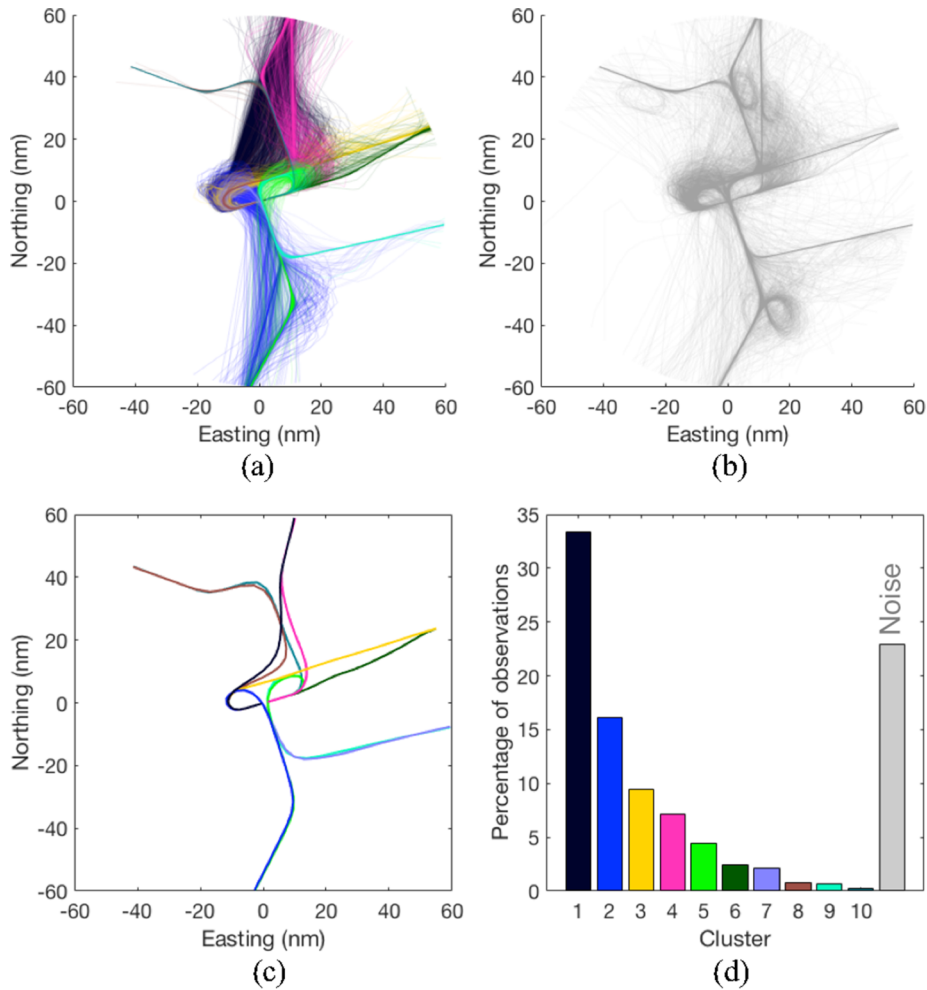


Fig. 7. Results of the trajectory clustering analysis for GRU arrivals. (a) Trajectory clusters; each color represents one cluster. (b) Trajectories labeled as noise. (c) Trajectory cluster centroids. (d) Distribution of trajectories by cluster.

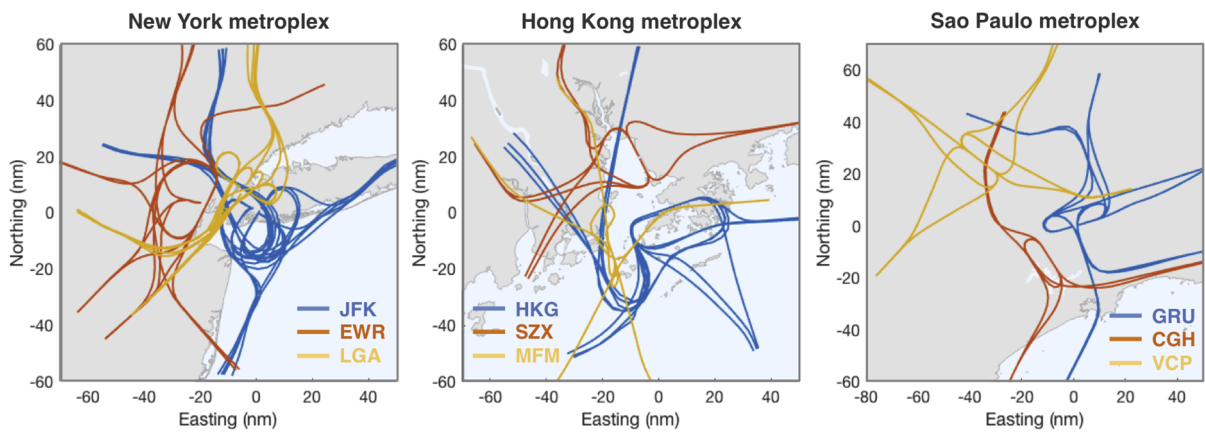


Fig. 8. Centroids of the metroplex arrival trajectory clusters.

accuracy higher than 98%, as shown in Table 3. The classifiers were used to assign trajectories in the remaining dataset (not used for the spatial clustering) to the learned routes and create flow matrices for each day of operations. As described in Section 3.2.2, the matrices were designed so that each element w_{ij} indicates the number of flights using a particular route i during a given hour j of the day. A flow was identified whenever $w_{ij} > 1$.

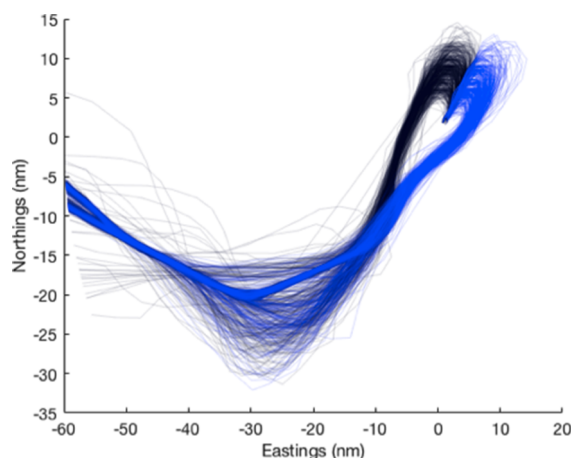


Fig. 9. Example of different trajectory patterns from arrival fix to runway threshold 22 at LGA.

Table 2

Number of trajectory tube intersections identified by pair of metroplex airports.

Multi-airport system	Airport pair	Number of trajectory tube intersections
New York	JFK – EWR	42
	JFK – LGA	130
	EWR – LGA	91
Hong Kong	HKG – SZX	12
	HKG – MFM	28
	SZX – MFM	30
Sao Paulo	GRU – CGH	53
	GRU – VCP	10
	CGH – VCP	16

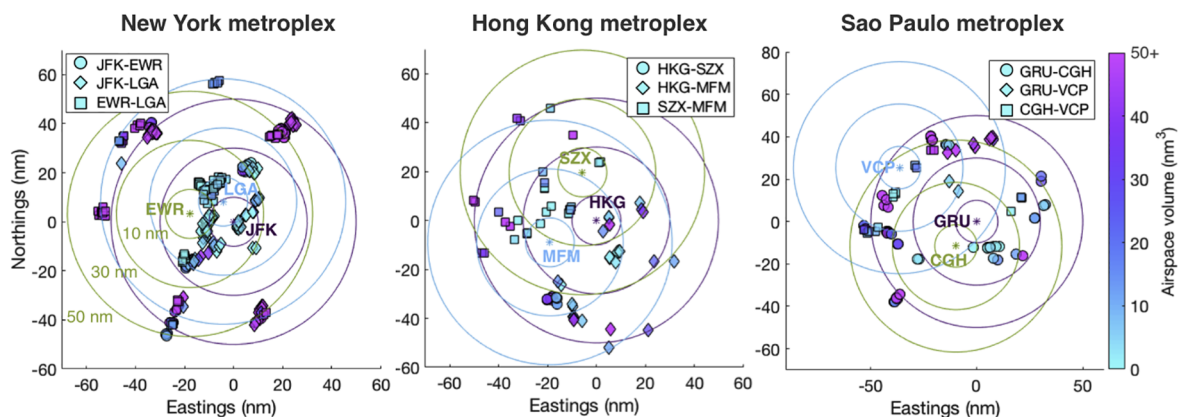


Fig. 10. Characterization of trajectory tube intersections.

In order to investigate the presence of patterns in the metroplex terminal area flow behavior, we used the third module of the trajectory data analytics framework and performed a second-layer clustering analysis on the set of time-dependent traffic flows R_t . For this, a data representation was established for the time-dependent metroplex flow structure with the Hourly Flow Vector FV_t and hierarchical clustering was used to partition the dataset of flow vectors for each multi-airport system. Table 4 summarizes the clustering results. It is noticeable that few clusters dominate and capture the majority of the observations, revealing the primary operational modes at each metroplex. We define a cluster of nominal flow structures as a *Metroplex Flow Pattern* (MFP). The New York metroplex stands out with the highest number of operational patterns and the highest operational variability. A set of thirty-three MFPs was identified for the New York metroplex, while only ten MFPs were identified for the Hong Kong metroplex and eight MFPs were identified for the Sao Paulo metroplex. The most frequent New York MFP accounted for only 16% of the observations. At the Hong Kong and Sao Paulo airport systems, the most frequent MFP accounted for 31% and 54% of the observations, respectively. The results suggest the New York metroplex is the most dynamic system. Indeed, it is observed that the New York terminal area has a

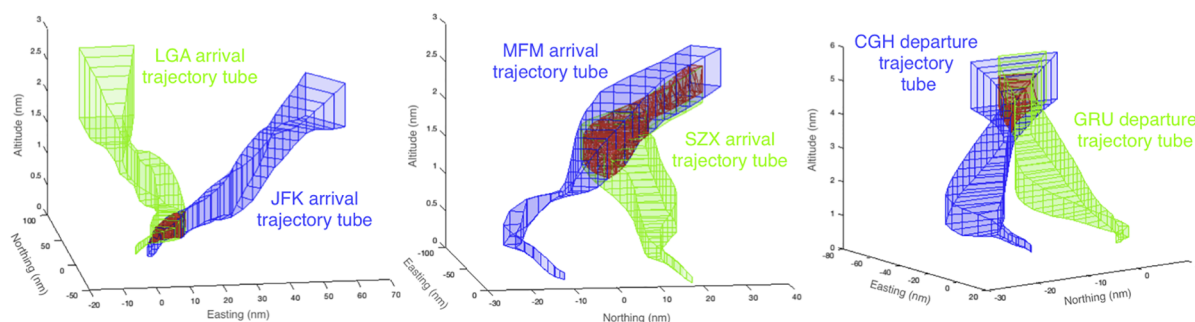


Fig. 11. Examples of trajectory tube intersections.

Table 3

Multi-way classification accuracy for trajectory classifiers.

Metroplex	Airport	Arrival trajectories	Departure trajectories
New York	JFK	99.0%	98.8%
	EWR	99.1%	98.8%
	LGA	98.8%	98.9%
Hong Kong	HKG	98.9%	99.2%
	SZX	98.7%	98.6%
	MFM	98.6%	98.4%
Sao Paulo	GRU	99.0%	99.0%
	CGH	99.1%	99.0%
	VCP	99.0%	98.9%

Table 4

Results of the metroplex traffic flow clustering analysis.

Multi-airport system	New York	Hong Kong	Sao Paulo
Total number of MFPs identified	33	10	8
Number of dominant MFPs that account for 85% of the observations	12	4	4
Frequency of occurrence of the most frequent MFP	16%	31%	54%
Median number of flow structure changes per day	5	2	2.5
Median dwell time of the most frequent MFP (h)	3.5	8	5

higher number of flow structure changes during the course of a day and the MFP dwell times are lower.

The top-3 MFPs in terms of frequency of occurrence at each multi-airport system are presented in Fig. 12. The average number of aircraft following each spatial trajectory pattern is displayed with different opacity levels (the maximum opacity corresponds to the maximum observed value) based on the members of each cluster. It is observed that the MFPs reflect different utilization patterns of runways and airspace. For example, Table 5 lists the airport runway configurations inferred for each MFP presented in Fig. 12. They are represented in the form “A | B”, where A indicates the arrival runways and B indicates the departure runways.

An investigation of the differences between the MFPs revealed some of the other factors that drive the behavior of the metroplex flows. Fig. 13 shows that the time of the day is an important variable determining the occurrence of MFPs across all three multi-airport systems. This was observed to be correlated with the specific daily demand patterns at each metroplex. For instance, it is noticeable that MFPs 2 and 3 in the New York metroplex are more likely to be observed in the morning and evening periods, which correspond to the usual bank of departures. Indeed, Table 5 shows that these MFPs are characterized by the use of multiple runways for departures at JFK.

We also observed a particular influence of meteorological conditions in the operational mode in the New York metroplex. Among the New York MFPs, five have more than 50% of their observations associated with periods of Instrument Meteorological Conditions (IMC). In the Hong Kong and Sao Paulo metroplexes, none of the MFPs showed such a high likelihood of being observed during IMC.

One of these five New York MFPs is shown in Fig. 14. For MFP 10, 80% of the observations are associated with periods of IMC. In the same figure, MFP 10 is contrasted with MFP 7, which was only observed during Visual Meteorological Conditions (VMC). It is noticed that the major difference between these MFPs is determined by the LGA arrival flows: in MFP 7, they are tailored to a visual approach to runway 31, whereas a long and straight Instrument Landing System (ILS) approach to the same runway is noticeable in MFP 10. In other words, part of the New York metroplex behavior is driven by the existence of multiple routes tailored to the approach procedure, with use governed by meteorological conditions.

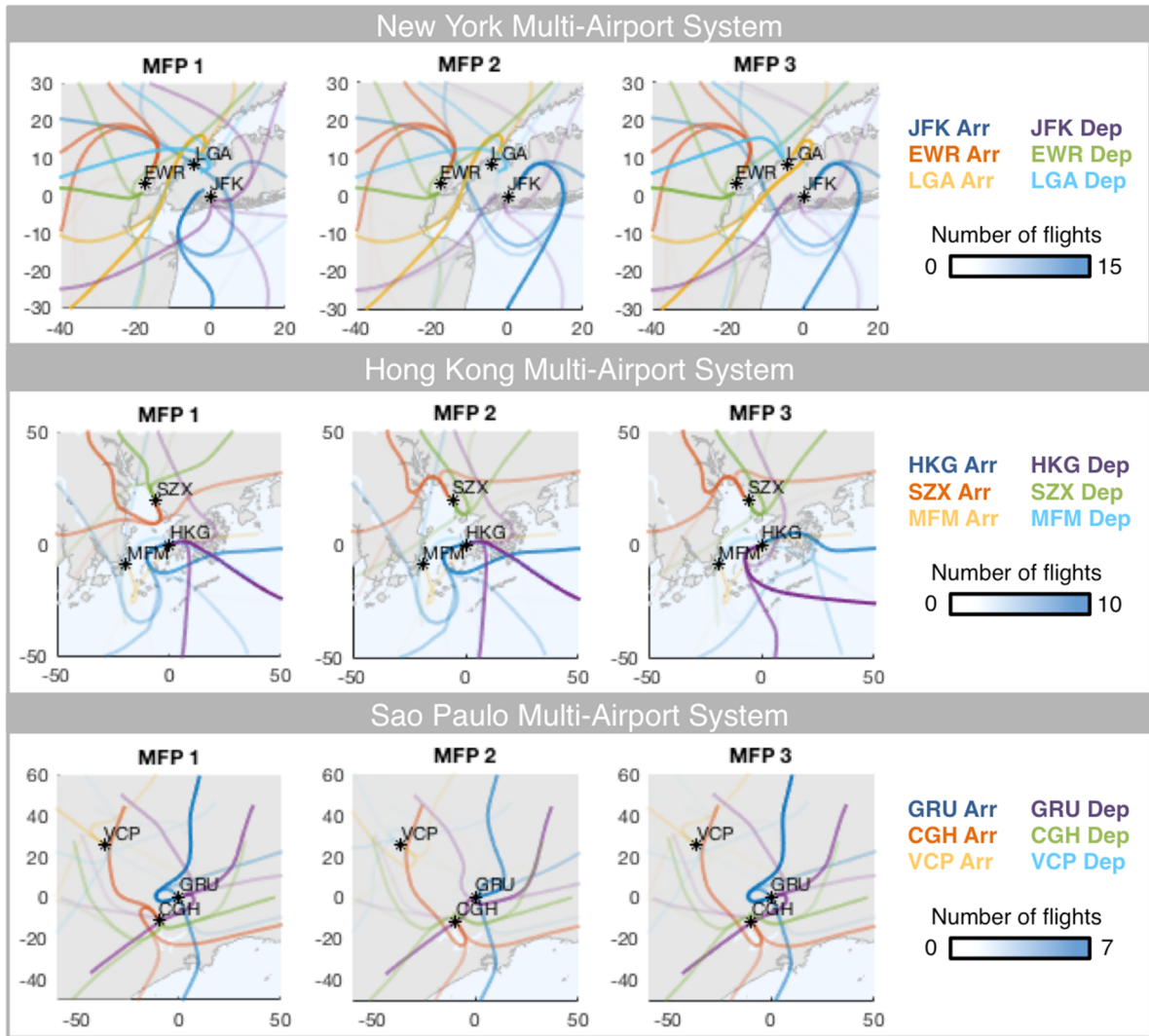


Fig. 12. Top-3 most frequently observed New York, Hong Kong and Sao Paulo MFPs.

Table 5

Airport runway configurations associated with each MFP.

Multi-airport system	Airport	MFP		
		1	2	3
New York	JFK	13L, 22L 13R	22L, 22R 22R, 31L	22L, 22R 22R, 31L
	EWR	22L 22R	22L 22R	22L 22R
	LGA	22 13	22 13	22 31
Hong Kong	HKG	07L 07R	07L 07R	25R 25L
	SZX	34 33	16 15	16 15
	MFM	34 34	34 34	34 34
Sao Paulo	GRU	09R 09L	27L 27R	09R 09L
	CGH	17R 17R	35L 35L	35L 35L
	VCP	15 15	33 33	15 15

5.2. Identification of metroplex flow interactions

We leveraged the knowledge about the flow patterns to identify relevant inter-airport flow interactions driven by the metroplex airspace design and use. The trajectory tube intersections identified in Section 4.2 can be generally translated into two types of flow interactions, depending on the use of the trajectory patterns involved in the intersection:

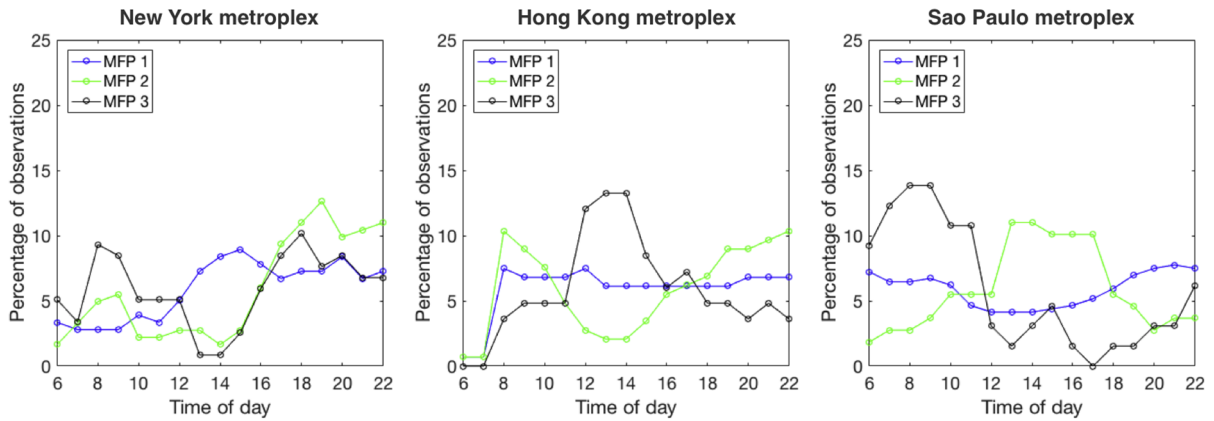


Fig. 13. MFP frequency of occurrence by time of day.

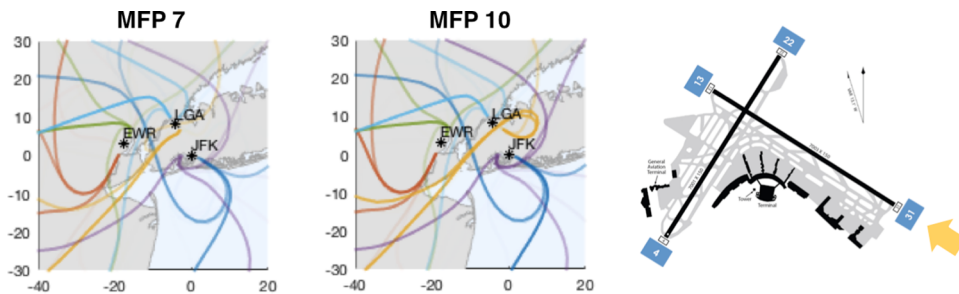


Fig. 14. Contrast between two New York MFPs in which the arrival flows are tailored to the approach procedure at LGA.

- **Flow dependency:** It occurs when the use of one trajectory pattern inhibits the use of the other one; therefore, flows are dependent and not observed simultaneously. From an operational perspective, a flow dependency is expected to make the management of runway/airspace configuration more challenging, as coordination is required between the airports.
- **Flow crossing:** It occurs when the use of one trajectory pattern does not inhibit the use of the other one; therefore, both flows can be observed simultaneously and they overlap at their shared airspace region. From an operational perspective, a flow crossing is expected to increase air traffic control complexity, as it tends to increase workload associated with conflict detection and avoidance in the shared airspace region.

We used the knowledge about the metroplex flow patterns to identify relevant flow crossings and flow dependencies as follows:

- A trajectory tube intersection is translated into a relevant *flow crossing* if there exists at least one MFP for which the trajectory patterns are used simultaneously and meaningfully. We define a meaningful use of a trajectory pattern if the median number of aircraft using the trajectory pattern is higher than one.
- A trajectory tube intersection is translated into a relevant *flow dependency* if, for all MFPs, the trajectory patterns are not used simultaneously. For this, we consider that whenever one trajectory pattern is used meaningfully (median number of aircraft is higher than one), the median number of aircraft using the other trajectory pattern should be equal to zero.

For the New York Metroplex, 60 flow dependencies were identified. For the Hong Kong and Sao Paulo metroplexes, flow dependencies were not observed. Fig. 15 shows the location of the trajectory tube intersections characterized as flow dependencies in the New York metroplex. JFK and LGA arrival flows were found to have the highest number of dependencies. Most of the flow dependencies are associated with shared airspace regions located approximately 10 nm from the airports. Clusters of trajectory tube intersections are observed close to the airports, revealing the existence of impracticable combinations of runway configurations. The results reveal that the New York metroplex is the most interdependent system, suggesting the management of runway and airspace configuration tends to be more challenging at this system.

Fig. 16 shows the median number of relevant flow crossings identified by MFP. The New York metroplex also has the highest number of flow crossings by MFP, revealing the typical terminal area flow structures are potentially characterized by higher levels of air traffic control complexity.

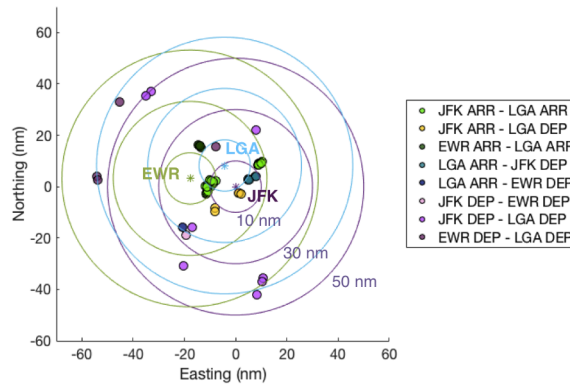


Fig. 15. Trajectory tube intersections associated with flow dependencies in the New York metroplex.

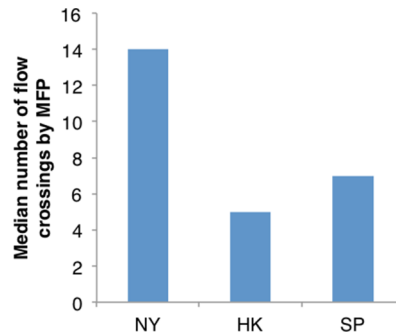


Fig. 16. Median number of flow crossings by MFP.

6. Characterization of performance differences

The performance analysis was focused on arrival operations, given that they are typically subject to higher levels of inefficiency as most of the arrival sequencing and scheduling is performed within the terminal area. We focused on the three performance areas of Efficiency, Capacity and Predictability, which are defined by the International Civil Aviation Organization (ICAO) as key areas for ATM performance measurement (ICAO, 2005).

6.1. Efficiency

In this performance area, we considered flight trajectory efficiency based on how actual trajectories compare to reference ideal trajectories. With this notion, we analyzed the efficiency of metroplex airspace design and use and the efficiency of the traffic flows.

6.1.1. Metroplex airspace design and use efficiency

We first compared the efficiency of the learned airspace structure for the three multi-airport systems in terms of trajectory lateral efficiency. Based on the trajectory patterns identified for each metroplex airport, we established a structural path stretch metric S , which is defined as the weighted average of the path stretch associated with the representative trajectory of each learned route:

$$S = \sum_{i=1}^N p_i s_i \quad (6)$$

Where S is the structural path stretch of an airspace with N trajectory patterns, p_i is the weight of trajectory pattern i and s_i is the path stretch associated with the centroid of trajectory pattern i , which is defined as the difference between the actual trajectory total length and the length of the shortest lateral path that connects the initial and the end points of the trajectory, assuming no aircraft performance, airspace or terrain restrictions. If the weights are equal for all trajectory patterns, the metric assesses the efficiency of metroplex airspace design regardless airspace use. If the weights are defined as the frequency of occurrence of trajectory patterns, the metric evaluates the combined efficiency of metroplex airspace design and use.

From the perspective of lateral performance, an airspace designed with lower structural path stretch can be considered more efficient as aircraft fly smaller distances between the terminal area boundary and the runway threshold. The structural path stretch calculated for arrival operations for each metroplex and each of their airports is shown in Fig. 17. The metric was calculated considering both equal weights and use-based weights.

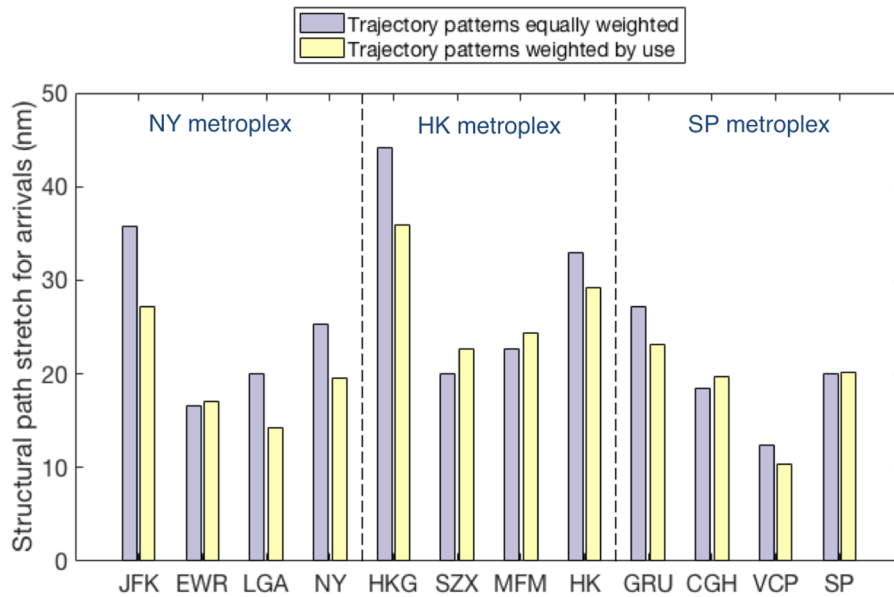


Fig. 17. Structural path stretch associated with the learned metroplex arrival route structure.

The Sao Paulo metroplex presents the most efficient airspace design, with the lowest structural path stretch. Interestingly, when airspace use is taken into account, the New York metroplex outperforms the others and shows the most efficient combination of airspace design and use. The metric is significantly higher for the Hong Kong metroplex. HKG stands out as the airport with the lowest efficiency, which is potentially due to the high level of de-confliction with the neighboring airports. At the other end, VCP stands out as the airport with the highest efficiency, ratifying the advantages of four-corner post airspace design.

6.1.2. Trajectory conformance

With the trajectory classification scheme, we also analyzed the lateral conformance of flight trajectories against the learned airspace structure. The Conformal Prediction framework was applied with a significance level $\varepsilon = 0.05$ to identify non-conforming behaviors. Fig. 18 shows the average daily percentage of non-conforming arrival trajectories at each metroplex airport. The results are displayed separately for days with fair weather conditions and for days with weather impacts. Overall, it is observed that the New York metroplex exhibits the best level of trajectory conformance, both for nominal and off-nominal conditions. HKG stands out as the airport with the highest percentage of non-conforming behaviors, well above average, even for nominal conditions. At the Sao Paulo metroplex, the primary airport GRU also shows the lowest level of trajectory conformance.

It is also observed that, for all airports, the percentage of non-conforming trajectories increases for days impacted by inclement meteorological conditions such as convection, low ceiling/visibility or strong winds. This suggests that weather is an important factor affecting the performance of terminal area operations at all three multi-airport systems. Indeed, Fig. 19 shows that weather impacted days are generally associated with increased levels of path stretching. As an example, the non-conforming trajectories identified for a convective weather day at JFK, HKG and GRU are shown in Fig. 20. It is observed that most of the trajectory deviations are caused by airborne holding, excessive vectoring or rerouting, in other words, tactical ATC instructions that tend to increase the length of the trajectory.

Finally, another interesting observation is that New York shows higher percentage increases in the level of path stretching during

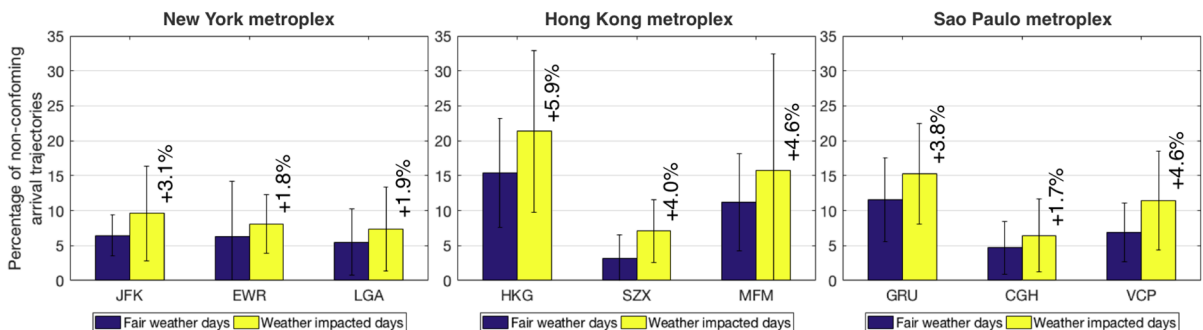


Fig. 18. Average daily percentage of non-conforming arrival trajectories.

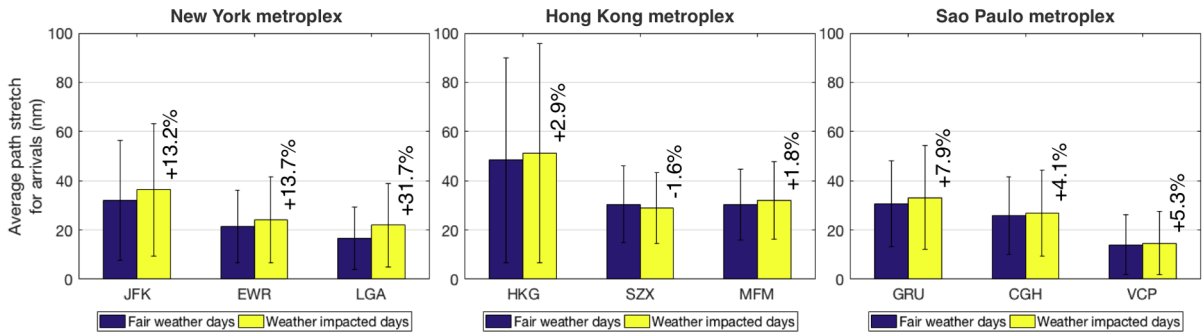


Fig. 19. Average path stretch for arrival trajectories.

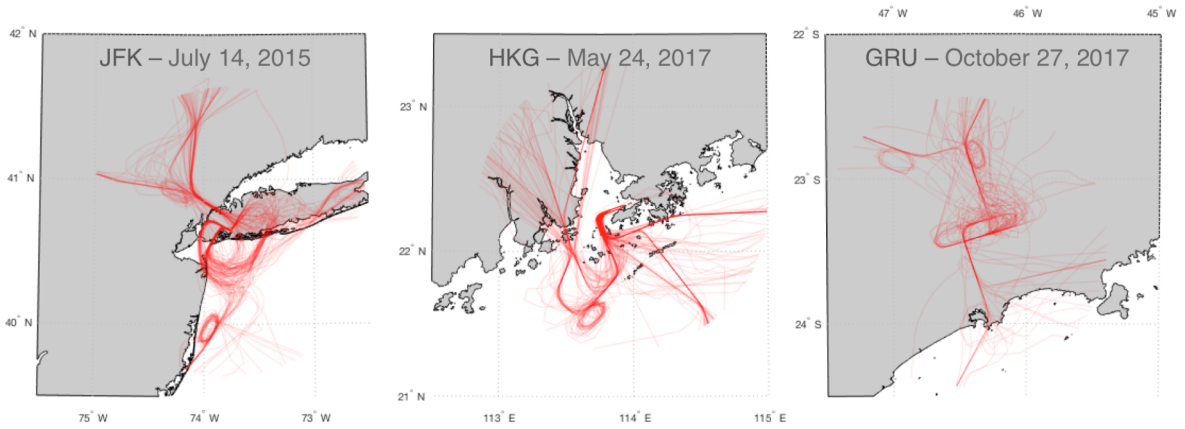


Fig. 20. Non-conforming arrival trajectories identified for a convective weather day at JFK, HKG and GRU.

inclement weather, especially LGA, revealing a more severe degradation in lateral performance, not only caused by trajectory deviations but also by the use of less efficient configurations (with high structural path stretch).

6.1.3. Traffic flow efficiency

We compared the overall traffic flow efficiency based on the efficiency of actual trajectories, both at spatial and temporal dimensions. Two metrics were defined. The lateral traffic flow efficiency is a distance-based efficiency metric. For a set of N flight trajectories associated with the traffic flow structure observed during a given time period, it is defined as:

$$Eff_{lateral} = \frac{\sum_{i=1}^N d_i}{\sum_{i=1}^N D_i} \quad (7)$$

where d_i is the length of the shortest path connecting the initial and end points of trajectory i and D_i is the actual length of trajectory i .

The temporal traffic flow efficiency is a time-based efficiency metric and it is defined in a similar fashion:

$$Eff_{temporal} = \frac{\sum_{i=1}^N t_i}{\sum_{i=1}^N T_i} \quad (8)$$

where t_i is the unimpeded flight time associated with trajectory i and T_i is the actual flight time associated with trajectory i . For each trajectory, its unimpeded flight time is estimated as the minimum between its actual flight time and the 10th percentile of the distribution of flight times for trajectories with the same spatial pattern.

Based on their definitions, both metrics range between 0 and 1. Values closer to one indicate higher efficiency. The lateral and temporal efficiencies were computed on an hourly basis for the metroplex flows. The median efficiency values for each metroplex are shown in Fig. 21. Overall, the New York metroplex presents the highest traffic flow efficiency, both spatially and temporally. The Hong Kong metroplex shows a significantly lower lateral efficiency, which is in part driven by its least efficient airspace design. It also presents the lowest temporal efficiency. Under inclement weather, however, the New York metroplex shows significant drops in efficiency, becoming the lowest performing system temporally.

We also calculated the median traffic flow efficiency for each of the dominant operational modes observed for each metroplex. Fig. 22 shows the distribution of the MFP efficiency values. While the New York metroplex on average outperforms the others both in terms of lateral and temporal efficiency, it exhibits the highest variability in MFP efficiency.

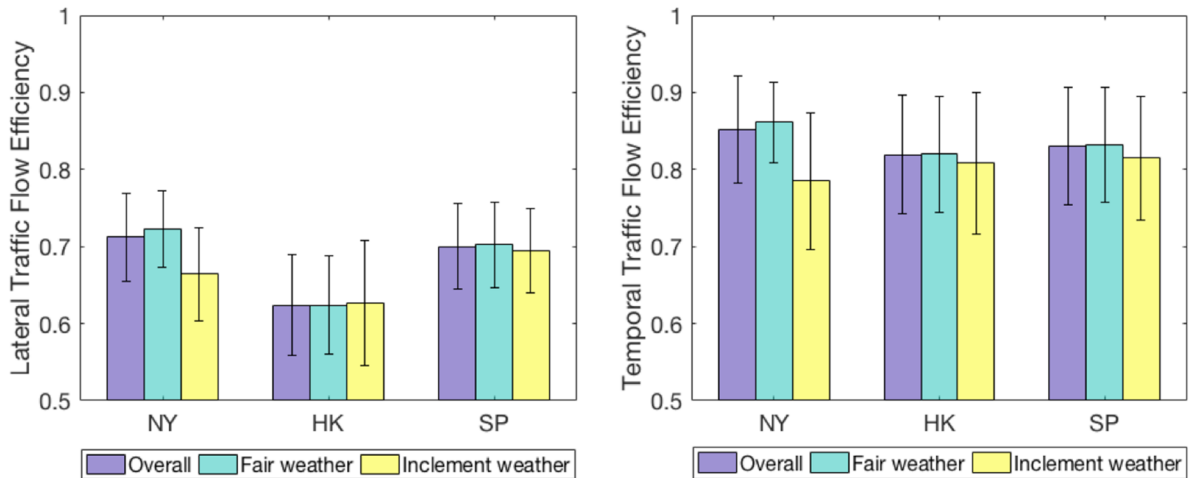


Fig. 21. Lateral and temporal traffic flow efficiency.

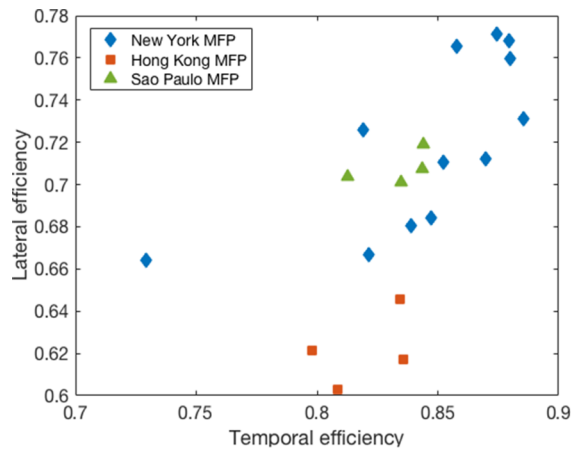


Fig. 22. Lateral and temporal traffic flow efficiency for the dominant MFPs at each metroplex.

6.2. Capacity

Capacity generally refers to an upper bound on the allowable throughput of a facility. We developed an empirical approach for assessing metroplex capacity from historical terminal area throughput and delay performance. Fig. 23(a,b) shows how the arrival throughput and the excess transit times in the New York terminal area change as a function of the arrival demand, under good weather conditions (VMC). Specifically, for every minute, the arrival throughput in the following 15-min period and the excess transit time for each flight that landed in the same 15-min period is plotted as a function of the number of arriving aircraft simultaneously present in the terminal area. Clearly, as the arrival demand increases, the arrival throughput increases in a disproportionate fashion until reaching a saturation level, i.e., point at which delivering more aircraft to the terminal area does not increase throughput because the system has reached its capacity. This behavior is reflected in the excess transit time curve. When the arrival demand increases, more aircraft are competing for the same resources and will encounter a higher probability of a delay assignment during the runway sequencing and scheduling process. When the saturation level is reached, the excess transit time increases much more rapidly.

Considering the observed throughput and delay profiles shown in Fig. 23, the empirical approach for metroplex capacity assessment was based on the estimation of performance curves that correlate arrival throughput and level of delay in the terminal area for each MFP. We formalized the estimation problem as a regression problem. We modeled the arrival throughput and the excess transit time in the terminal area with a piece-wise linear function of the arrival demand (number of arriving aircraft in the terminal area), with additional constraints arising from expected operational behavior: the arrival throughput was modeled as a non-decreasing and concave function of the arrival demand and the excess transit time was modeled as a non-decreasing function of the arrival demand. The parameters of the curves were estimated using quantile regression (Koenker and Hallock, 2001). Only VMC observations were used in the estimation process in order to marginalize out weather related throughput impacts and enable a cleaner assessment of metroplex configuration performance.

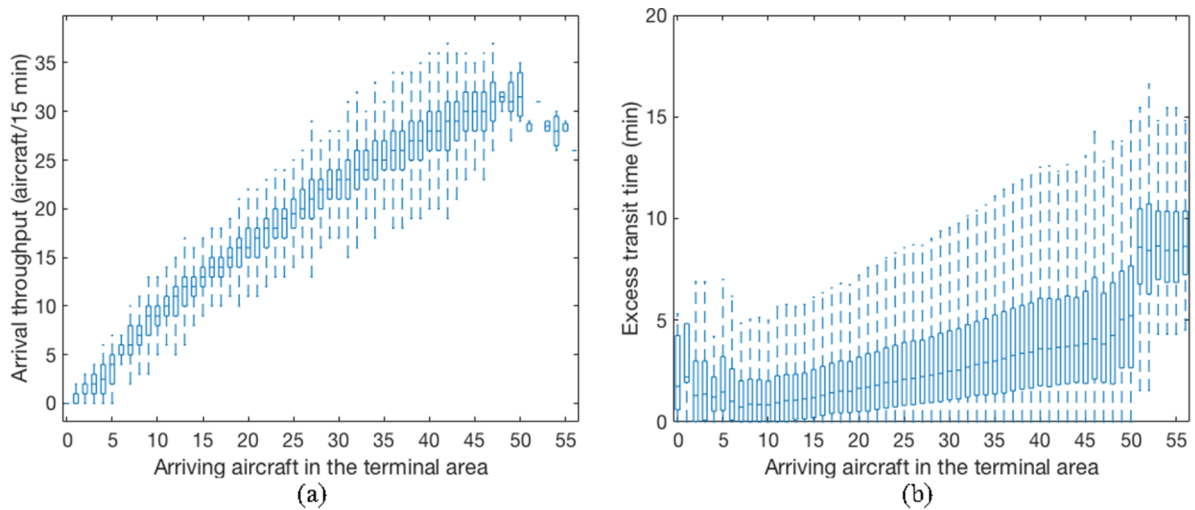


Fig. 23. (a) Arrival throughput as a function of the arrival demand in the terminal area for the New York metroplex. (b) Excess terminal area transit time as a function of the arrival demand in the terminal area for the New York metroplex.

With the performance curves obtained with a median regression fit for the dominant MFPs at each metroplex, we extracted their saturation capacity as well as the level of delay associated with the saturation capacity. The maximum throughput normalized by the number of active arrival runways as well as the level of terminal area delay associated with the maximum throughput are presented in Fig. 24 for each MFP at each metroplex. The results reveal that New York is the highest throughput system, followed by Hong Kong and Sao Paulo. The median arrival rate per runway is 30 aircraft/h, 20 aircraft/h and 16 aircraft/h at the New York, Hong Kong and Sao Paulo metroplexes, respectively. Yet, the New York metroplex stands out with the largest standard deviation in throughput performance, emphasizing its operational variability, as also discussed in the previous section. The highest arrival rate per runway is 34 aircraft/h, while the lowest is 23 aircraft/h. The median level of delay at saturation is 3.7 min, but the standard deviation is also high. At the Hong Kong and Sao Paulo metroplexes, smaller variation in throughput performance is observed. For Hong Kong, the median level of delay at saturation is 4.4 min. Sao Paulo stands out with a relatively higher median delay level of 6.4 min.

6.3. Predictability

Predictability is defined by ICAO as the ability of airspace users to provide consistent and dependable levels of performance (ICAO, 2005). It is typically assessed by measuring the variation in system performance experienced by the users (CANSO, 2015). In this section, we discuss interrelations between ATM configuration predictability and ATM performance predictability.

Previously, we observed that New York presents the highest variability in metroplex configuration performance, both in terms of lateral and temporal traffic flow efficiency and of arrival throughput (Figs. 22 and 24). The New York metroplex also shows the

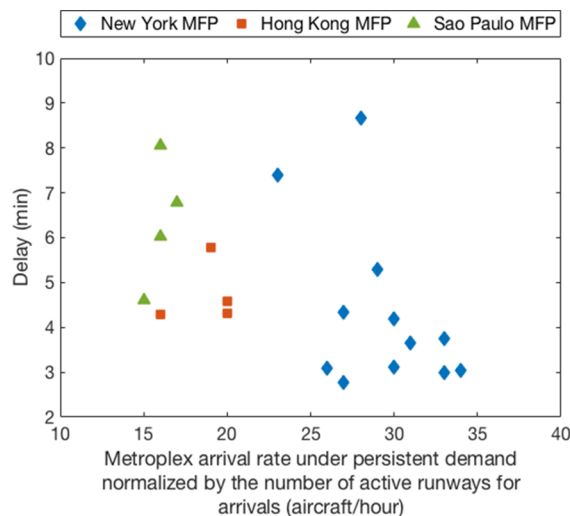


Fig. 24. Metroplex arrival rates under persistent demand and associated level of delay.

lowest MFP dwell times and the highest number of changes in the flow structure per day (Table 4), suggesting the ATM configuration is less predictable in this system. We translated this variation in ATM configuration into daily capacity variation in order to analyze how the ATM configuration predictability affects ATM performance predictability.

The average accumulated change in capacity per day was calculated for each metroplex. For this, we considered the observed MFPs for each day and their associated maximum achievable throughput (normalized by the number of runways), as shown in Fig. 24. The New York system was found to present the highest capacity variation per day of 11 aircraft. For Hong Kong and Sao Paulo, a small capacity variation of less than 2 aircraft was observed. From an operational perspective, it means the New York system tends to be more reliant on traffic flow management in order to adjust demand with the more dynamic system capacity.

7. Discussion

The characterization of terminal area traffic flows in the New York, Hong Kong and Sao Paulo metroplexes revealed structural and operational differences between these multi-airport systems. First, the New York metroplex was observed to have a more complex route design, with a higher number of routes and a higher number of intersections between them. A significant number of these intersections were found to be located close to the airport and were translated into flow dependencies, revealing the existence of impracticable combinations of runway configuration and routes. On the other hand, for the Hong Kong and Sao Paulo systems, most of the inter-airport route intersections were found to be located further from the airports and were characterized by larger airspace volumes resulting from the sharing of both airspace fixes and path segments.

The analysis also showed the New York system is the one with more dynamic changes in the terminal area flow behavior during daily operations. While the metroplex traffic flows could be categorized in few patterns in the Hong Kong and Sao Paulo systems, they showed much higher variability in the New York system. This variability can be attributed to a couple of factors: higher variability in the set of runway configurations by airport; presence of inter-airport flow dependencies that create operationally infeasible combinations of runway configuration and routes; higher airspace design complexity that enables ATM to tailor operations for visual and instrument meteorological conditions. The New York metroplex was also found to be the most interdependent system, with a significant number of flow dependencies, and the typical terminal area flow structures were found to have a higher number of flow crossings, suggesting higher levels of air traffic control complexity.

By analyzing the impacts of such structural and operational differences on performance, we found the more complex New York airspace presents the most efficient design and use in terms of trajectory lateral performance. The Hong Kong airspace design showed a high level of structural path stretch, which is potentially the cost of its more decoupled airspace, with fewer flow interactions with the neighboring airports. On average, the New York metroplex showed the highest traffic flow efficiency, both spatially and temporally. Yet, it exhibited the highest variability in traffic flow efficiency as well as more pronounced drops in efficiency during inclement weather. The Hong Kong metroplex showed a significantly lower lateral traffic flow efficiency, which was observed to be a result of not only its least efficient airspace design, but also of tactical traffic flow management, as revealed by its lower temporal traffic flow efficiency and trajectory conformance. In terms of capacity, the New York metroplex was found to be the highest throughput system, but stood out with the largest variability in throughput performance. Differences in metroplex arrival capacity across MFPs were as great as 44 aircraft per hour. Such differences were associated not only with its more diverse set of runway configurations, but also with differences in the terminal area flow structure.

The more dynamic New York metroplex flow behavior, particularly influenced by meteorological conditions, suggests the coupled ATM configuration in this multi-airport system as well as its performance might be less predictable. Indeed, we observed the New York system exhibited the highest variability in traffic flow efficiency and capacity by MFP. From an operational perspective, this means the New York system is more reliant on traffic flow management in order to adjust demand with the more dynamic system capacity. At the same time, it also means traffic flow management tends to become more challenging, since it has to deal with increased levels of uncertainty in the ATM configuration and associated constraints.

Overall, the results highlighted different areas of action to be considered at each multi-airport system towards improving their terminal area operations. The high level of structural path stretch in the Hong Kong metroplex, particularly for HKG, revealed opportunities for improvement in the airspace structure in order to increase trajectory lateral efficiency. Enhancements in airspace design in the Hong Kong and Sao Paulo metroplexes might also be performed to de-couple routes from different airports that share the same airspace region in order to achieve a more expeditious flow. The lower level of trajectory conformance and temporal efficiency in the Hong Kong and Sao Paulo metroplexes also revealed opportunities for improvement in tactical traffic flow management towards better sequencing and scheduling of arriving flights. Finally, the higher level of airport interdependency, dynamism, weather dependency and performance variability in the New York metroplex revealed opportunities for improving the airspace design in order to de-conflict flows close to the airports and for integrating weather forecasting with decision-making in order to better anticipate weather conditions, translate them into operational impact and plan traffic flow management efficiently.

8. Conclusions

This paper presents a flight trajectory data analytics framework for identifying spatial and temporal patterns in aircraft movement and providing a high-fidelity characterization of air traffic flows. The framework includes three modules: clustering flight trajectories at spatial scale, trajectory classification, and clustering air traffic flows at temporal scale. Different machine learning techniques are especially incorporated into the three modules to process aircraft trajectory data and enable the characterization of traffic flows.

We demonstrated the use of this data analytics framework with a comparative analysis of terminal area operations between the

New York, Hong Kong and Sao Paulo multi-airport systems. With a systematic descriptive approach to analyze metroplex airspace design and use and to assess operational performance, we investigated structural, operational and performance differences between these three multi-airport systems. We found that the New York multi-airport system presents the most complex airspace design and the most dynamic flow behavior. Interestingly, it exhibited the best levels of trajectory lateral and temporal efficiency on average, yet the highest variability in operational performance. By contrast, the Hong Kong and Sao Paulo multi-airport systems showed a relative simple and clean airspace design and lower variability in traffic flow behavior and operational performance.

The results also highlighted different areas of action to be considered at each multi-airport system towards improving their air traffic operations. The Hong Kong metroplex route structure was found to be considerably less efficient, particularly for HKG, revealing opportunities for improvement in airspace design in order to increase trajectory lateral performance. The lower level of trajectory conformance and temporal efficiency observed for the Hong Kong and Sao Paulo metroplexes revealed opportunities for improvement in tactical traffic flow management towards better sequencing and scheduling of arriving flights. Sao Paulo was found to be the lowest throughput performing system, emphasizing the importance of actions to expand capacity at the individual airports. Finally, the higher level of airport interdependency and operational variability observed in the New York metroplex revealed opportunities for improving the airspace design in order to reduce flow interdependencies and provide more consistent all-weather operations. The particular influence of weather conditions on airspace use and traffic performance in the New York metroplex also highlighted the importance of integrating weather forecasting with decision-making in order to better anticipate weather conditions, translate them into operational impact and plan traffic flow management efficiently at this system.

The flight trajectory data analytics framework was demonstrated at the terminal area scale for three multi-airport systems in this paper, but it can be easily applied to other terminal areas to obtain a deeper understanding of the structure and dynamics of use of the airspace and to allow for cross-comparisons of operational performance. Another clear direction for future research is the application of the framework for characterization of air traffic flows at other scales. For instance, characterizing national-level air traffic flows can generate useful insights about en route traffic behavior for supporting en route air traffic management. One might be interested in exploiting routing patterns between origin and destination to diagnose en route flight inefficiencies, to better understand the factors that drive the selection of a particular route by flight operators, to discover feasible rerouting options or to assess the impacts of adverse weather conditions (such as convective weather) on route acceptability and capacity. At the sector level, characterization of traffic flow patterns may be useful for complexity management applications. One might be interested in exploiting patterns of air traffic control complexity in order to predict periods of low/high complexity and better inform staffing or sector reconfiguration decisions. While the methods incorporated in the framework have general applicability, adaptations should be needed (e.g., data representation for clustering) to tailor the analysis for the intended application. Ultimately, the knowledge about the traffic flows and their performance derived from trajectory data analytics can provide the basis for developing new strategies, procedures and decision support tools for air traffic management.

Conflict of interest statement

There are no conflicts of interest.

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