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Optimal preventive maintenance strategy for leased equipment under successive usage-based contracts

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In the context of equipment leasing, maintenance service is usually bundled with the leased equipment and offered by the lessor as an integrated package under a lease contract. The lessor is then responsible to prescribe an effective maintenance policy to keep the equipment operational in an economical way. This paper investigates upgrade and preventive maintenance (PM) strategies for industrial equipment during successive usage-based lease contracts with consideration of a warranty period, from the lessor's perspective. The accelerated failure time model and age reduction model are adopted to capture the effect of usage rate and imperfect PM/upgrade on the equipment reliability, respectively. More importantly, since equipment usage rates may vary across different lease contracts, this study develops an age correspondence framework to characterise usage rate shifts between successive lease periods. The optimal upgrade degree and the optimal number and level of PM actions are progressively updated for each upcoming lease period to minimise the total expected lease servicing cost, by considering the usage rate and maintenance implementation history. Numerical studies show that under given cost structures, periodical PM activities within each lease period tends to outperform the pre-leasing upgrade actions, though both of them can reduce the lease servicing cost.

Keywords: Successive leasing; usage-based contract; warranty; upgrade; maintenance management; cost analysis

1. Introduction

1.1. Background and motivation

Businesses require various types of equipment to manufacture their products or to provide services. For instance, mining companies need excavators and dump trucks to load and transport mining materials. However, rapid technological obsolescence and increased complexity of equipment, coupled with high cost of ownership, make leasing instead of owning equipment an economical option (Nisbet and Ward 2001), especially for small- and medium-sized companies. Meanwhile, due to the lack of specialised maintenance tools and well-trained maintenance crews, equipment lessees tend to outsource maintenance activities to lessors (Murthy and Jack 2014). In this context, maintenance service is usually bundled with the leased equipment and offered by the lessor as an integrated package under a lease contract (Xia et al. 2018). As such, prescribing an effective maintenance policy, especially preventive maintenance (PM), in the lease contract is under the responsibility of the lessor, and it is of vital importance to both the lessee and the lessor.

This paper is interested in the maintenance optimisation for leased industrial equipment such as cranes, excavators, and dump trucks, from the lessor's perspective. For such equipment, there are three important characteristics in their leasing problem that should be considered during the determination of an optimal maintenance programme:

(i) *Warranty contracts.* Nowadays, almost all products are sold with warranty contracts, and industrial equipment is no exception (He et al. 2017; Xie, Shen, and Zhong 2017; Zhao, He, and Xie 2018). During the warranty period, any eligible equipment failures are entitled to be rectified by the original equipment manufacturer (OEM). When planning PM schedule for industrial equipment under lease, it is necessary to take the OEM warranty policy into account.
(ii) *Usage-based lease contracts.* Industrial equipment often operates under harsh conditions, and the equipment reliability is closely related to its usage. In practice, the lease contracts for many industrial equipment are characterised by both lease duration and usage. This type of contract is called usage-based lease contract (Hamidi,

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Liao, and Szidarovszky 2016). It is worth mentioning that the usage-based lease contract is a special case of the two-dimensional lease contract in Iskandar and Husniah (2017), with the usage limit being infinity.

(iii) *Successive leasing*. The useful life of most industrial equipment is quite long (typically 10–25 years). In this case, a piece of equipment can be leased multiple times within its useful life, since a single lease period is usually much shorter than such a long useful life. When a lease contract ceases, the lessor retains ownership of the equipment and can renew the lease contract if the lessee is interested or lease the same equipment to a new lessee (Murthy and Pongpech 2008). This is referred to as successive leasing. In fact, such successive leasing of the same equipment is the core business of many leasing companies that specialise in leasing specific equipment to various lessees (Ben Mabrouk, Chelbi, and Radhoui 2016b). In the successive leasing context, two essential aspects should be considered: (*a*) during a specific lease period, the information about the lease durations and usage rates of future lease contracts is often unavailable; and (*b*) when a piece of equipment is leased more than once (i.e. it becomes a used one), its condition needs to be inspected and upgrade procedures may be required to improve its reliability (Pongpech, Murthy, and Boondiskulchock 2006).

The three characteristics above raise a challenge to the lessors, namely, how to plan appropriate maintenance schedules for leased industrial equipment under successive usage-based contracts. The existing approaches and models in the literature are far from sufficient for assisting the lessors in making such decisions. The present paper intends to bridge this gap.

1.2. Literature review

In the literature, the optimisation of PM strategies for leased equipment has been reported extensively. Jaturonnatee, Murthy, and Boondiskulchok (2006) made an early attempt to study a general PM policy for leased equipment. Yeh and Chang (2007) investigated the optimal threshold value of failure rates for leased products with PM actions. Yeh, Chang, and Lo (2011) jointly studied the optimal lease duration and PM policy for leased equipment to maximise the lessor's expected profit. Schutz and Rezg (2013) proposed two maintenance strategies for leased equipment to meet a minimum reliability requirement of the lessee. Moreover, a multi-phase PM policy for leased equipment was developed in Zhou et al. (2015), in which the lease period was divided into multiple phases and different PM frequencies were applied in different phases. Zhou et al. (2016) investigated a periodical PM strategy for leased equipment subject to continuous internal degradation and stochastic external shock damage, simultaneously. Ben Mabrouk, Chelbi, and Radhoui (2016a) studied a PM policy for leased equipment in a context where both PM actions and corrective repairs were imperfect. Hung, Tsai, and Chang (2017) incorporated random failure penalties into the PM contract of leased equipment, and adopted the Black-Scholes equation to characterise the expected revenue losses (i.e. the penalty) stemming from overdue repair. Recently, Xia et al. (2017) developed a lease-oriented opportunistic maintenance methodology for multi-unit leased systems under product-service paradigm. Some earlier literature on this topic can be referred to Chang and Lo (2011), Pongpech and Murthy (2006), Pongpech, Murthy, and Boondiskulchock (2006), Yeh, Chang, and Lo (2010), and Yeh, Kao, and Chang (2009), among others.

The studies above focused predominately on optimising various PM policies within a single time-based lease period. Despite the importance of warranty contract, only three papers incorporated the warranty contract into the optimisation of PM policies for leased equipment, i.e. Hajej, Rezg, and Gharbi (2015, 2016) and Ben Mabrouk, Chelbi, and Radhoui (2016b). Nevertheless, all the three papers considered lessor warranty within the lease period, instead of the OEM warranty. For the usage-based lease contracts, Hamidi, Liao, and Szidarovszky (2016) was the only work addressing this issue. They developed a game-theoretic model, under which the lessee determined the optimal lease duration and usage rate while the lessor was responsible for identifying an optimal PM policy. Furthermore, Ben Mabrouk, Chelbi, and Radhoui (2016b) was the sole research that studied the successive leasing problem. In their work, they implicitly assumed perfect information regarding future lease durations, and then determined optimal upgrade levels between successive leasing periods to maximise the lessor's profit over the equipment lifecycle.

Our literature review reveals that the aforementioned three characteristics are either ignored or separately considered in the literature, and currently no research that integrates all of them in the context of industrial equipment leasing, has been reported. This paper contributes to the literature by developing an integrated framework to optimise upgrade and PM strategies under successive usage-based lease contracts considering the OEM warranty.

1.3. Overview

In this paper, both upgrade and PM actions are considered for industrial equipment under successive lease. More specifically, pre-leasing upgrade actions are carried out between successive lease periods; while imperfect PM actions are periodically performed within each lease period, and the PM frequency can be altered only when a new lease period starts. The effect of usage rate and imperfect PM/upgrade on the equipment reliability are captured by the accelerated failure time (AFT) model and age reduction model, respectively. Since the equipment usually operates under different usage rates during different lease periods, a statistical-virtual-age-based age correspondence framework is developed in this work. Furthermore, since the information regarding future lease contracts may be unavailable in the successive leasing context, the ideal case, in which the optimal PM schedule over the whole lifecycle is identified with pre-known lease durations and usage rates, is not practical. Thus, this paper proposes to progressively update the PM schedule for each upcoming lease period considering the usage rate and the upgrade/PM implementation history. Numerical studies show that both upgrade and PM strategies are beneficial to the lessor in terms of lease cost reduction, while the periodical PM activities within each lease period tend to be more cost-efficient than the pre-leasing upgrade procedures.

The remainder of this paper is organised as follows. Section 2 formulates the model elements, including the equipment failure model, the upgrade model, and the imperfect PM model. Then, Section 3 develops and analyzes the optimisation model of progressive upgrade and PM updating strategy. Section 4 presents numerical examples and sensitivity analyses to demonstrate the maintenance optimisation model. Finally, Section 5 concludes this paper and presents several topics for future research.

2. Model formulation

2.1. Problem description

Consider that a lessor purchases pieces of new industrial equipment sold with a two-dimensional warranty (W, U), where W and U are time and usage limits, respectively. The OEM commits to rectify any equipment failures during the warranty period. This warranty contract expires when either the equipment age exceeds W or the total usage reaches U, whichever occurs first. This policy is commonly adopted for industrial equipment such as cranes, excavators, and dump trucks (Ye and Murthy 2016; Wang and Xie 2018). It should be noted that the warranty policy is often determined by the OEM considering obligation or market competition, and is thus treated as exogenous from the lessor's perspective.

Within the useful life of a piece of equipment, the lessor earns revenue by sequentially renting it to various lessees. When a lease contract ceases, the equipment is returned to the lessor, who upgrades it to a better status. Then, the lessor may renew the lease contract if the lessee is interested (the renewed contract is not necessarily identical to the original one) or lease it to a new lessee. Denote the parameters of the jth usage-based lease contract by (L_j, r_j), where L_j is the length of the lease period and r_j is the usage rate of the equipment (see Figure 1). Note that, since leasing market is typically lessee-oriented, we suppose that from the lessor's perspective, L_j and r_j are deterministic quantities and their values are pre-specified by the lessee.

Under a lease contract, the corrective maintenance of the equipment is billed to the lessor, unless the equipment is protected by the OEM warranty. Besides, each equipment failure over the lease period would incur a penalty cost to the lessor because a failure will result in downtime and thus production loss for the lessee. In this work, we consider both upgrade and PM activities implemented to mitigate equipment degradation, and thus to reduce the total lease servicing cost. The upgrade action is applied at the very beginning of each lease period (except the first one), while imperfect PM actions are periodically performed within each lease period. In this manner, the decision variables for the jth lease period are the upgrade level q_j ($q_1 = 0$), the number of PM actions n_j, and the corresponding PM level m_j (see Figure 1).

It is necessary to point out that due to the successive leasing manner mentioned above, the lessor should progressively update the maintenance schedule based on the upcoming lease contract and the upgrade/PM implementation history during previous lease periods. Another thing noteworthy is that in some countries, industrial equipment such as trucks should be mandatorily discarded after certain years, say, T_{life}, or after certain usage, say, U_{life}, whichever occurs first. In this case, before signing the jth lease contract, the lessor has to check whether the equipment will run out of its useful life or not, at

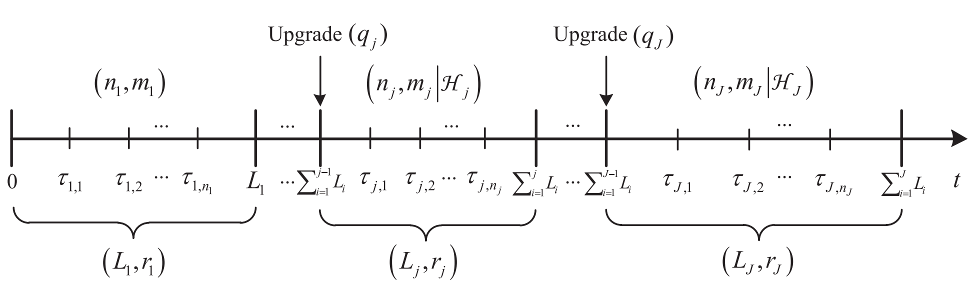

Figure 1. The framework of upgrade and PM during successive lease periods.

the end of this lease period. In other words, the equipment can be leased for the jth contract if and only if

$$\sum_{i=1}^{j} L_i \le T_{life} \quad \text{and} \quad \sum_{i=1}^{j} L_i r_i \le U_{life}. \tag{1}$$

Since the lessor does not have the information regarding future contracts in advance, the maximal number of lease periods J is known only when the proceeding condition (1) is violated. That is to say,

$$J = \max\left\{ j \,\middle|\, \sum_{i=1}^{j} L_i \le T_{life} \quad \text{and} \quad \sum_{i=1}^{j} L_i r_i \le U_{life} \right\}. \tag{2}$$

2.2. *Modelling equipment failures*

Generally speaking, industrial equipment gradually degrades with age and usage. It is necessary to incorporate the effect of usage rate into the equipment failure model, especially for the usage-based contracts. In this study, the marginal approach is employed to model the failure intensity $\lambda(t \mid r)$ in terms of time t and usage rate r, in which the usage rate is treated as a covariate (Jack, Iskandar, and Murthy 2009; Wang and Xie 2018). Specifically, a well-known variation of the marginal approach, namely, the AFT model, is adopted here; see Iskandar and Husniah (2017), Tong, Song, and Liu (2017), and Ye et al. (2013), for reference.

An assumption adopted here is that the equipment usage rate is constant within any single lease period, but it may vary across different lease periods. Further assume that before signing a lease contract, the lessor is able to know the equipment usage rate in this single period in advance, which is specified by the lessee. The two assumptions above are appropriate for industrial equipment. For example, excavators in mining plants normally operate in a routine manner, say, eight hours a day. In this case, the usage rate can be tentatively identified before signing a specific lease contract. Nevertheless, usage rates of the same equipment during different lease periods may be different, depending on the types and loads of the duties.

Consider a piece of equipment designed for some nominal usage rate r_0. If T_r [T_0] denotes the time to first failure under usage rate r [r_0], then using the AFT formulation we have (Jack, Iskandar, and Murthy 2009)

$$\frac{T_r}{T_0} = \left(\frac{r_0}{r}\right)^{\gamma},$$

where $\gamma > 0$ is the acceleration factor.

Furthermore, if the distribution function and failure rate function of T_0 are given by $F_0(t)$ and $\lambda_0(t)$, respectively, then under usage rate r, the distribution function and failure rate function of T_r can be expressed as

$$F(t \mid r) = F_0\left(t\left(\frac{r}{r_0}\right)^{\gamma}\right), \tag{3}$$

and

$$\lambda(t \mid r) = \frac{f(t \mid r)}{1 - F(t \mid r)} = \left(\frac{r}{r_0}\right)^{\gamma} \lambda_0\left(t\left(\frac{r}{r_0}\right)^{\gamma}\right), \tag{4}$$

where $f(t \mid r) = \mathrm{d}F(t \mid r)/\mathrm{d}t$. The parameters of $F(t \mid r)$ and $\lambda(t \mid r)$ can be estimated from field failure data and/or warranty claim data (Dai et al. 2017; Yang, He, and He 2016). Interested readers are referred to Wang and Xie (2018) and Wu (2013) for a summary of statistical methodologies of two-dimensional warranty data analysis.

In this study, any failures within the lease periods are assumed to be rectified through minimal repairs. The minimal repair assumption is quite appropriate for complex industrial equipment where an equipment failure occurs due to a component failure and the equipment can be made operational by replacing the failed component with a new identical one (Murthy 1991). As a consequence, the equipment failure rate after repair is nearly the same as that just before failure. Under this assumption, it is well known that equipment failures over time will occur according to a non-homogeneous Poisson process (NHPP) with the intensity function having the same form as the failure rate for the time to first failure, i.e. $\lambda(t \mid r)$.

2.3. *Modelling upgrade and PM actions*

In this study, pre-leasing upgrade and post-leasing PM actions are simultaneously considered to improve the equipment reliability for better operational performance. This section presents the modelling and analysis of the upgrade and imperfect PM actions in detail.

2.3.1. Modelling imperfect PM actions

In practice, a PM action might include a set of maintenance tasks such as cleaning, lubricating, adjusting/calibrating, systematic inspection, and/or component replacements (Ben Mabrouk, Chelbi, and Radhoui 2016b; Zhao et al. 2018). The impact of such a PM action on the equipment reliability is usually imperfect, which corresponds to imperfect PM. In this work, the discrete PM modelling framework in Kim, Djamaludin, and Murthy (2004) is employed to characterise the effect of imperfect PM actions. This discrete PM model has been frequently adopted by many studies; see Ben Mabrouk, Chelbi, and Radhoui (2016b), Darghouth, Chelbi, and Ait-kadi (2017), Su and Wang (2016b), and Wang and Su (2016), for example.

Assume that the duration required to perform a PM action is very short comparing to a lease period, thus is negligible. Let n_j denote the number of PM actions applied within the jth lease period, then the corresponding PM interval is $\Delta_j = L_j/(n_j+1), j = 1, 2, \ldots, J$. Meanwhile, denote the kth, $k = 1, 2, \ldots, n_j$, PM instant during the jth lease period by $\tau_{j,k}$, then we have

$$\tau_{j,k} = \sum_{i=1}^{j-1} L_i + k\frac{L_j}{n_j+1}, \quad j = 1, 2, \ldots, J;\ k = 1, 2, \ldots, n_j, \tag{5}$$

with $\tau_{1,0} = 0$, $\tau_{j,n_j+1} = \tau_{j+1,0} = \sum_{i=1}^{j} L_i$ for $j = 1, 2, \ldots, J$ (see Figure 1).

The underlying assumption adopted here is that a PM action rejuvenates the equipment so that it can effectively reduce the equipment's virtual age. The amount of age reduction is assumed to be proportional to the supplement of equipment age since the last PM action (Kijima 1989; Kim, Djamaludin, and Murthy 2004). As a result, within the jth lease period, the virtual age immediately after the kth PM action is modelled as

$$v_{j,k} = v_{j,k-1} + \delta(m_j)(\tau_{j,k} - \tau_{j,k-1}), \quad j = 1, 2, \ldots, J;\ k = 1, 2, \ldots, n_j, \tag{6}$$

where $m_j \in [0, M]$ is the level of PM effort within the jth lease contract, and $\delta(m_j)$ is the corresponding age reduction factor.

A PM level m_j corresponds to a specific set of maintenance tasks, and the corresponding $\delta(m_j)$ can be estimated from historical maintenance data. Notice that, larger value of m_j indicates a greater PM effort, and thus $\delta(m_j)$ is a decreasing function with respect to m_j. More specifically, if $m_j = 0$, then $\delta(0) = 1$, which implies that there is no PM; if $m_j = M$ (impossible to achieve in practice), then $\delta(M) = 0$, which means that this model does not allow PM effect to be perfect; while if $m_j \in (0, M)$, then the PM action is imperfect. In this study, we assume $\delta(m_j) = (1 + m_j)e^{-m_j}$, as in Kim, Djamaludin, and Murthy (2004).

2.3.2. Modelling upgrade actions

As mentioned earlier, when a piece of equipment is leased more than once, it is no longer a new equipment (but a used one). In this case, before re-leasing, upgrade effort may be required to improve its initial reliability status to a certain extent. Although upgrade strategies for used equipment have attracted much attention in the area of warranty management (Diallo et al. 2017; Khatab, Diallo, and Sidibe 2017; Su and Wang 2016a; Wang, Xie, and Li 2018), this topic still needs more investigations in the successive leasing context. In this work, we consider imperfect upgrade actions to be performed between successive lease periods, along with the periodical PM actions within each lease period.

Here, the impact of an imperfect upgrade action is also described by the age reduction approach (Shafiee and Chukova 2013; Wang, Xie, and Li 2018). Let $\hat{v}_{j,0}$, $j = 2, 3, \ldots, J$, denote the equipment virtual age at the very beginning of the jth lease period (i.e. before upgrade), which will be elaborated later. The effect of an imperfect upgrade action lies in reducing the equipment virtual age from $\hat{v}_{j,0}$ to $v_{j,0} = (1 - q_j)\hat{v}_{j,0}$, $q_j \in [0, 1]$. In this manner, a larger q_j represents a greater upgrade effort, which corresponds to a 'younger' equipment virtual age after upgrade.

Following Pongpech, Murthy, and Boondiskulchock (2006), the upgrade cost is modelled as an increasing function of q_j, and is given by

$$C_u(q_j) = \frac{C_s q_j \hat{v}_{j,0}}{1 - \exp\left\{-\varphi \hat{v}_{j,0}(1 - q_j)\right\}}, \quad j = 2, 3, \ldots, J, \tag{7}$$

where $C_s > 0$ and $\varphi > 0$.

Note that, if $q_j = 0$ (i.e. no upgrade), then $C_u(0) = 0$; if $q_j \to 1$, then $C_u(q_j) \to \infty$, which implies that it is economically not possible to upgrade a used equipment to an as-good-as-new state. Notice also that, there is no need to carry out an upgrade action at the beginning of the first lease period, as the equipment is new. Thus, q_1 is set to zero, and $C_u(q_1) = 0$.

2.3.3. Age correspondence framework

Since industrial equipment often operates under different usage rates over different lease periods, an age correspondence framework is developed in this section to characterise the usage rate shifts between successive lease periods.

By combining the upgrade and PM effects together, $v_{j,k}$ in (6) can be further derived as

$$v_{j,k} = \left(1 - q_j\right) \hat{v}_{j,0} + k\delta(m_j)\Delta_j, \quad j = 1, 2, \ldots, J;\ k = 1, 2, \ldots, n_j, \tag{8}$$

with $v_{1,0} = 0$ and $q_1 = 0$.

In Equation (8), $\hat{v}_{j,0}$ is closely related to the equipment's virtual age at the end of the $(j-1)$th lease period, which is

$$\begin{aligned} v_{j-1,n_{j-1}+1} &= v_{j-1,n_{j-1}} + \Delta_{j-1} \\ &= \left(1 - q_{j-1}\right) \hat{v}_{j-1,0} + n_{j-1}\delta(m_{j-1})\Delta_{j-1} + \Delta_{j-1}, \quad j = 2, 3, \ldots, J. \end{aligned} \tag{9}$$

Since the equipment is supposed to operate under different usage rates during the $(j-1)$th and the jth lease periods, $v_{j-1,n_{j-1}+1}$ and $\hat{v}_{j,0}$ are generally not identical, unless $r_{j-1} = r_j$. To establish an age correspondence for the equipment under different usage rates, the concept of *statistical virtual age* in Finkelstein (2007) is adopted in this study. With this concept, operating the equipment for $v_{j-1,n_{j-1}+1}$ time units under usage rate r_{j-1} is equivalent to operating it for $\hat{v}_{j,0}$ time units under usage rate r_j (Wang and Xie 2018). From the reliability perspective, this statement corresponds to

$$F\left(v_{j-1,n_{j-1}+1} \mid r_{j-1}\right) = F\left(\hat{v}_{j,0} \mid r_j\right), \quad j = 2, 3, \ldots, J. \tag{10}$$

Substituting (3) and (9) into (10), the equipment virtual age (before upgrade) at the very beginning of the jth, $j = 2, 3, \ldots, J$, lease period, i.e. $\hat{v}_{j,0}$, can be expressed as

$$\begin{aligned} \hat{v}_{j,0} &= v_{j-1,n_{j-1}+1}\left(\frac{r_{j-1}}{r_j}\right)^{\gamma} \\ &= \left[\left(1 - q_{j-1}\right) \hat{v}_{j-1,0} + n_{j-1}\delta(m_{j-1})\Delta_{j-1} + \Delta_{j-1}\right]\left(\frac{r_{j-1}}{r_j}\right)^{\gamma}, \quad j = 2, 3, \ldots, J. \end{aligned} \tag{11}$$

Accordingly, one can easily obtain $v_{j,k}$ by substituting (11) into (8).

Remark 1 As can be seen from Equation (11), $\hat{v}_{j,0}$ decreases as the upgrade level q_{j-1}, the number of PM actions n_{j-1}, and/or the corresponding PM level m_{j-1} increases. A lower initial virtual age means that the equipment is statistically

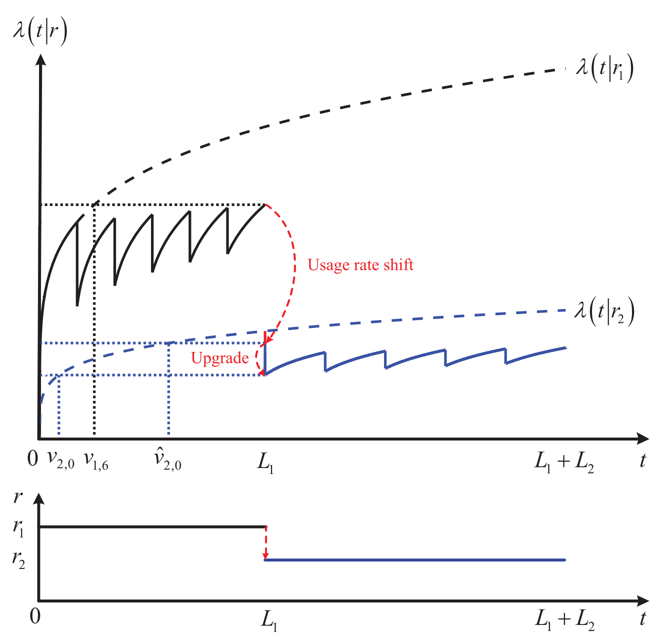

Figure 2. Illustration of the effects of upgrade/PMs and usage rate shifts on the intensity function.

'younger', and it may require less upgrade and PM services during subsequent lease periods. This is the way that the upgrade/PM implementation history $\mathcal{H}_j = (q_1 = 0, n_1, m_1; \ldots; q_{j-1}, n_{j-1}, m_{j-1})$ affects the maintenance optimisation during subsequent lease periods.

For illustrative purpose, Figure 2 visualises the effects of upgrade/PM actions and usage rate shift on the virtual age. The periodical virtual age reductions within each single lease period are due to the effect of imperfect PM actions; while the virtual age is updated twice between the first and the second lease periods, which stems from the shift of usage rates and the upgrade action, respectively. Figure 2 shows that operating the equipment for $v_{1,6}$ time units under usage rate r_1 corresponds to operating it for $\hat{v}_{2,0}$ time units under usage rate r_2. From (11), we have $v_{1,6} < \hat{v}_{2,0}$ since $r_1 > r_2$. It is worth mentioning that the area under the curve $\lambda(t \mid r_1)$ from 0 to $v_{1,6}$ is identical to that under the curve $\lambda(t \mid r_2)$ from 0 to $\hat{v}_{2,0}$ (Hu, Jiang, and Liao 2017). Furthermore, the upgrade action reduces the equipment's virtual age from $\hat{v}_{2,0}$ to $v_{2,0}$, which effectively improves the equipment reliability.

3. The optimisation model

In this section, the upgrade and PM optimisation model of industrial equipment under successive lease periods is developed. The lessor's problem is to progressively determine the optimal upgrade and PM strategy for the jth, $j = 1, 2, \ldots, J$, lease period in order to minimise the expected lease servicing cost within this period.

As mentioned before, within the warranty period, only failure penalty cost is incurred to the lessor; while after the warranty expires, the lessor have to bear both the repair cost and the penalty cost. Thus, the warranty term (W, U) has an important effect on the lessor's total lease servicing cost. For the sake of simplicity, hereafter, we only consider the case where the actual warranty length is shorter than the first lease period, i.e. $\min\{W, U/r_1\} \le L_1$. It is not difficult to extend our optimisation model to a general case, i.e. the OEM warranty terminates within the jth, $j = 1, 2, \ldots, J$, lease period, which is briefly introduced in Appendix.

3.1. The first lease period

During the first lease period, n_1 PM actions are periodically carried out with constant interval $\Delta_1 = L_1/(n_1 + 1)$. Since equipment failures are minimally repaired with negligible durations, the failure process between any two successive PM actions follows an NHPP. Thus, the expected number of equipment failures within the first lease period is given by

$$E[N_1(n_1, m_1)] = \sum_{k=0}^{n_1} \int_{v_{1,k}}^{v_{1,k}+\Delta_1} \lambda(t \mid r_1)\, \mathrm{d}t, \tag{12}$$

where $v_{1,k}$ is given by (8).

Moreover, given that the actual warranty length W_r is shorter than the first lease period, i.e. $W_r = \min\{W, U/r_1\} \le L_1$, the number of PM actions within the warranty period is $d = \max\{k \mid k\Delta_1 < W_r\}$. The expected number of equipment failures within $(W_r, L_1]$ can thus be obtained as

$$E\left[N_1^{\Phi}(n_1, m_1)\right] = \int_{v_{1,d}+W_r-d\Delta_1}^{v_{1,d}+\Delta_1} \lambda(t \mid r_1)\, \mathrm{d}t + \sum_{k=d+1}^{n_1} \int_{v_{1,k}}^{v_{1,k}+\Delta_1} \lambda(t \mid r_1)\, \mathrm{d}t.$$

Essentially, in the first lease period, the total cost to the lessor is comprised of the following four elements:

(i) *Minimal repair cost.* As mentioned before, the minimal repair cost for equipment failures over the period $(W_r, L_1]$ is borne by the lessor. Given that the average cost of a minimal repair is C_f, the total expected minimal repair cost is $C_f E[N_1^{\Phi}(n_1, m_1)]$.

(ii) *PM cost.* As there are n_1 PM actions within the first lease period, the total PM cost is $n_1 C_p(m_1)$, where $C_p(m_1)$ is the average cost of a PM with level m_1.

(iii) *Type I penalty cost.* The Type I penalty cost is incurred to the lessor if the failed equipment is not restored to an operational state within a reasonable time (Jaturonnatee, Murthy, and Boondiskulchok 2006). Let Y_i denote the repair duration of the ith failure, $1 \le i \le N_1(n_1, m_1)$. Then, the total Type I penalty cost will be $C_t \sum_{i=1}^{N_1(n_1,m_1)} \max\{0, Y_i - \bar{t}\}$, where C_t is the Type I penalty cost per unit time and $\bar{t}$ is a pre-set repair time threshold. Hence, the total expected cost associated with Type I penalty can be derived as $C_t E[N_1(n_1, m_1)] \int_{\bar{t}}^{\infty} [1 - G(y)]\mathrm{d}y$, where $G(y)$ is the distribution function of the repair time.

(iv) *Type II penalty cost.* If the number of equipment failures exceeds a pre-set threshold $\bar{n}$, a fixed penalty cost C_n for each additional failure would be incurred to the lessor since this would result in production loss for the lessee (Jaturonnatee, Murthy, and Boondiskulchok 2006). Without loss of generality, we consider $\bar{n} = 0$ in this work, which is reasonable for most industrial equipment. In this case, the total expected cost associated with Type II penalty is $C_n E[N_1(n_1, m_1)]$.

As a result, the lessor's total expected servicing cost within the first lease period $[0, L_1]$ is the sum of these four elements, and is given by

$$\begin{aligned} E\left[C_1(n_1, m_1)\right] &= C_f E\left[N_1^{\Phi}(n_1, m_1)\right] + n_1 C_p(m_1) + \left(\hat{C}_t + C_n\right) E\left[N_1(n_1, m_1)\right] \\ &= C_f \left[\int_{v_{1,d}+W_r-d\Delta_1}^{v_{1,d}+\Delta_1} \lambda\left(t \mid r_1\right)\, \mathrm{d}t + \sum_{k=d+1}^{n_1} \int_{v_{1,k}}^{v_{1,k}+\Delta_1} \lambda\left(t \mid r_1\right)\, \mathrm{d}t\right] \\ &\quad + \left(\hat{C}_t + C_n\right) \sum_{k=0}^{n_1} \int_{v_{1,k}}^{v_{1,k}+\Delta_1} \lambda\left(t \mid r_1\right)\, \mathrm{d}t + n_1 C_p\left(m_1\right), \end{aligned} \tag{13}$$

where $\hat{C}_t = C_t \int_t^{\infty} [1 - G(y)]\, \mathrm{d}y$.

The lessor's optimisation problem for the first lease period is thus to determine the optimal number and degree of PM actions (i.e. n_1^* and m_1^*) to minimise the total expected lease servicing cost, as follows.

$$\min_{n_1, m_1} E[C_1(n_1, m_1)] \text{ s.t. } n_1 \in Z^+ \cup \{0\}, \quad 0 \le m_1 \le M. \tag{14}$$

It is difficult, if not impossible, to obtain closed-form solution to the optimisation problem (14). Fortunately, both n_1 and m_1 are integers, so simple numerical search methods are efficient to identify the optimal solution.

3.2. *The jth ($j \geq 2$) lease period*

In this section, the optimisation model for the jth, $j = 2, 3, \ldots, J$, lease period is presented, which is different from that of the first period.

After the expiration of the $(j-1)$th lease contract, the lessor would rent the same equipment to a new (or the same) lessee with negotiated lease length L_j and usage rate r_j. Before re-leasing, the equipment is subject to an upgrade action to recover its intrinsic reliability, and the upgrade level is q_j. During the jth lease period, the lessor carries out n_j periodical PM actions with level m_j. Then, the expected number of equipment failures is given by

$$E\left[N_j\left(q_j, n_j, m_j \mid \mathcal{H}_j\right)\right] = \sum_{k=0}^{n_j} \int_{v_{j,k}}^{v_{j,k}+\Delta_j} \lambda(t \mid r_j)\, \mathrm{d}t, \quad j = 2, 3, \ldots, J, \tag{15}$$

where $v_{j,k}$ is given by (8) and $\mathcal{H}_j = (q_1^* = 0, n_1^*, m_1^*; \ldots; q_{j-1}^*, n_{j-1}^*, m_{j-1}^*)$, $j = 2, 3, \ldots, J$.

Since the lessor has to bear the upgrade cost, PM cost, minimal repair cost, and failure penalty costs during the jth, $j = 2, 3, \ldots, J$, lease period, the total expected lease servicing cost can be derived as

$$\begin{aligned} E\left[C_j\left(q_j, n_j, m_j \mid \mathcal{H}_j\right)\right] &= \left(C_f + \hat{C}_t + C_n\right) E\left[N_j\left(q_j, n_j, m_j \mid \mathcal{H}_j\right)\right] + C_u(q_j) + n_j C_p(m_j) \\ &= \left(C_f + \hat{C}_t + C_n\right) \sum_{k=0}^{n_j} \int_{v_{j,k}}^{v_{j,k}+\Delta_j} \lambda(t \mid r_j)\, \mathrm{d}t + \frac{C_s q_j \hat{v}_{j,0}}{1 - \exp\left\{-\varphi \hat{v}_{j,0}(1 - q_j)\right\}} + n_j C_p(m_j). \end{aligned} \tag{16}$$

Therefore, the lessor's optimisation problem for the jth lease contract is to determine the optimal upgrade level, the optimal number and level of PM actions (i.e. q_j^*, n_j^*, and m_j^*) to minimise the total expected lease servicing cost, as follows.

$$\min_{q_j, n_j, m_j} E\left[C_j\left(q_j, n_j, m_j \mid \mathcal{H}_j\right)\right] \text{ s.t. } q_j \in [0, 1], n_j \in Z^+ \cup \{0\},\ 0 \le m_j \le M, \quad j = 2, 3, \ldots, J. \tag{17}$$

As can be seen from (17), as the lessor sequentially leases the same equipment to a series of lessees, the optimal upgrade and PM policy should be progressively updated for an upcoming lease period j based on the contract information (L_j,

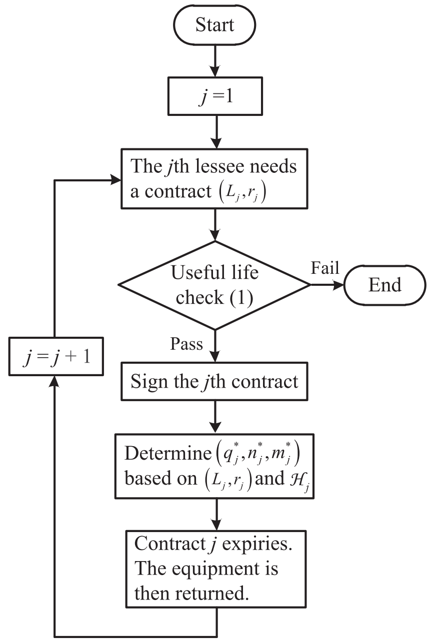

Figure 3. The progressive updating framework of maintenance optimisation.

r_j) and the maintenance history $\mathcal{H}_j$. This progressive nature is due to the unavailability of future contract information in the successive leasing context. In summary, the proposed maintenance optimisation framework is illustrated in Figure 3.

After the last lease period, the total expected servicing cost within the whole lifecycle of the equipment can be determined as follows.

$$E\left[TC\left(\mathbf{q}^*, \mathbf{n}^*, \mathbf{m}^*\right)\right] = \sum_{j=1}^{J} E\left[C_j\left(q_j^*, n_j^*, m_j^* \mid \mathcal{H}_j\right)\right],$$

where $\mathbf{q}^* = (q_1^* = 0, q_2^*, \ldots, q_J^*)$, $\mathbf{n}^* = (n_1^*, n_2^*, \ldots, n_J^*)$, and $\mathbf{m}^* = (m_1^*, m_2^*, \ldots, m_J^*)$.

3.3. Model analysis

3.3.1. Special cases

Two special cases of the proposed upgrade and PM strategy are discussed, namely, only upgrade strategy and only PM strategy, respectively.

(a) $n_j = 0$ and $m_j = 0$ (i.e. only upgrade strategy). In this case, only upgrade action is performed at the beginning of each lease period. Then, the lessor's total expected lease servicing cost at the jth lease period is given by

$$E\left[C_j\left(q_j, 0, 0 \mid \mathcal{H}_j\right)\right] = \begin{cases} C_f \int_{W_r}^{L_1} \lambda(t \mid r_1)\,\mathrm{d}t + \left(\hat{C}_t + C_n\right) \int_0^{L_1} \lambda(t \mid r_1)\,\mathrm{d}t, & j = 1, \\ \left(C_f + \hat{C}_t + C_n\right) \int_{v_{j,0}}^{v_{j,0}+L_j} \lambda(t \mid r_j)\,\mathrm{d}t + \dfrac{C_s q_j \hat{v}_{j,0}}{1 - \exp\left\{-\varphi \hat{v}_{j,0}(1 - q_j)\right\}}, & j = 2, \ldots, J, \end{cases} \tag{18}$$

where $v_{j,0} = (1 - q_j)\hat{v}_{j,0} = (1 - q_j)[(1 - q_{j-1})\hat{v}_{j-1,0} + L_{j-1}](r_{j-1}/r_j)^{\gamma}, j = 2, 3, \ldots, J$.

(b) $q_j = 0$ (i.e. only PM strategy). In this case, periodical PM actions are carried out within each lease period, while no upgrade actions are performed between successive lease periods. Then, the lessor's total expected lease servicing cost at the

jth lease period is given by

$$E\left[C_j(0,n_j,m_j \mid \mathcal{H}_j)\right] = \begin{cases} C_f\left[\int_{v_{1,d}+W_r-d\Delta_1}^{v_{1,d}+\Delta_1} \lambda(t \mid r_1)\,\mathrm{d}t + \sum_{k=d+1}^{n_1}\int_{v_{1,k}}^{v_{1,k}+\Delta_1} \lambda(t \mid r_1)\,\mathrm{d}t\right] \\ \quad + \left(\hat{C}_t + C_n\right)\sum_{k=0}^{n_1}\int_{v_{1,k}}^{v_{1,k}+\Delta_1} \lambda(t \mid r_1)\,\mathrm{d}t + n_1 C_p(m_1), & j = 1, \\ \left(C_f + \hat{C}_t + C_n\right)\sum_{k=0}^{n_j}\int_{v_{j,k}}^{v_{j,k}+\Delta_j} \lambda(t \mid r_j)\,\mathrm{d}t + n_j C_p(m_j), & j = 2,\ldots,J, \end{cases} \tag{19}$$

where $v_{j,0} = \hat{v}_{j,0} = [\hat{v}_{j-1,0} + n_{j-1}\delta(m_{j-1})\Delta_{j-1} + \Delta_{j-1}](r_{j-1}/r_j)^{\gamma}, j = 2, 3, \ldots, J$.

3.3.2. *When is an upgrade action beneficial?*

It is beneficial for the lessor to apply an upgrade procedure if the total expected leasing cost with both upgrade and PM actions is less than that with only PM actions ($q_j = 0$). Let n_j^{o} and m_j^{o} denote the optimal number and level of imperfect PM actions such that $E[C_j(0, n_j, m_j \mid \mathcal{H}_j)]$ in (19) is minimised. Then, the upgrade procedure is beneficial if

$$E\left[C_j\left(q_j^*, n_j^*, m_j^* \mid \mathcal{H}_j\right)\right] < E\left[C_j\left(0, n_j^{\mathrm{o}}, m_j^{\mathrm{o}} \mid \mathcal{H}_j\right)\right], \quad \text{for } j = 2, 3, \ldots, J.$$

3.3.3. *When are PM actions beneficial?*

It is beneficial for the lessor to implement scheduled PM actions if the total expected leasing cost with both upgrade and PM actions is less than that with only upgrade actions ($n_j = 0$ and $m_j = 0$). Let $q_j^{\mathrm{o}} = 0$ denote the optimal upgrade degree such that $E[C_j(q_j, 0, 0 \mid \mathcal{H}_j)]$ in (18) is minimised. Then, the PM actions are beneficial if

$$E\left[C_j\left(q_j^*, n_j^*, m_j^* \mid \mathcal{H}_j\right)\right] < E\left[C_j\left(q_j^{\mathrm{o}}, 0, 0 \mid \mathcal{H}_j\right)\right], \quad \text{for } j = 1, 2, \ldots, J.$$

4. Numerical examples and sensitivity analyses

In this section, numerical examples are presented to demonstrate the applicability of the proposed maintenance optimisation model and strategy. Also, detailed sensitivity analyses are performed on key model parameters to study the effect of input uncertainty on the results.

4.1. Numerical study

Here the industrial equipment of interest are excavators made in China, which are sold with a warranty period of $W = 12$ months and $U = 2000$ hours, whichever occurs first. The excavator failures under nominal usage rate r_0 follow a two-parameter Weibull distribution with scale parameter α and shape parameter β. Thus, using the AFT formulation (4), the failure intensity under actual usage rate r can be derived as

$$\lambda(t \mid r) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}\left(\frac{r}{r_0}\right)^{\gamma\beta}. \tag{20}$$

It is known that the excavators are designed to operate under nominal usage rate $r_0 = 0.167 \times 10^3$ hours/month. Based on warranty data analysis, the model parameters in (20) are obtained as $\alpha = 1.24$, $\beta = 1.20$, and $\gamma = 3$ (Yang, He, and He 2016).

Within the useful life of an excavator, the lessor earns revenue by successively renting it to various lessees. Under a lease contract, an excavator failure incurs an average minimal repair cost of $C_f = \$100$ to the lessor. In addition, each excavator failure would result in a Type II penalty cost of $C_n = \$100$. The repair time of a failure follows Weibull distribution $G(y)$, with scale and shape parameters being 0.5 and 0.5, respectively, so that the mean repair time is one hour. If the repair time exceeds $\bar{t} = 2$ hours, then there is a penalty $C_t = \$300$ per additional hour. As a result, periodical imperfect PM actions during each lease period are carried out by the lessor to improve the excavator reliability and thus reduce the repair and penalty expenses. We consider $\delta(m_j) = (1 + m_j)e^{-m_j}$ for $m_j = 0, 1, \ldots, 5$, and the corresponding PM costs are $C_p(0) = \$0$,

$C_p(1) = \$10$, $C_p(2) = \$30$, $C_p(3) = \$60$, $C_p(4) = \$100$, and $C_p(5) = \$160$, as in Kim, Djamaludin, and Murthy (2004). At the same time, upgrade procedure is implemented at the beginning of each lease period to further improve the excavator's health status. The parameters of the upgrade cost are $C_s = 10$ and $\varphi = 0.01$. In this numerical study, the units for time and usage rate are months and 1000 hours/month, respectively, while the unit for cost is US dollar ($), unless noted specifically.

In this example, we consider three successive lease contracts $(L_1, r_1) = (36, 0.151)$, $(L_2, r_2) = (48, 0.130)$, and $(L_3, r_3) = (30, 0.173)$. For instance, the third lease contract lasts for 30 months and the negotiated usage rate is 173 hours/month (approximately 8 hours per working day). The grid search method is then adopted to obtain the optimal upgrade level and the optimal number and level of PM actions in each lease period. It should be mentioned that $q_j^*, j = 2, 3, \ldots, J$, is searched with a step of 0.01. The computation time is quite short in this setting (several seconds). In this manner, the solution quality and computation speed are good enough for practical use.

Based on the parameter settings above, the optimal upgrade and/or PM decisions under the three maintenance strategies are listed in Table 1. The proposed strategy produces optimal solutions $(n_1^*, m_1^*) = (6, 5)$, $(q_2^*, n_2^*, m_2^*) = (0.12, 4, 4)$, and $(q_3^*, n_3^*, m_3^*) = (0.47, 6, 4)$ for the three lease periods, respectively, with the total expected lifecycle lease cost being \$36771.7. For illustrative purpose, Figure 4 shows the existence of the optimal maintenance decisions for the second lease period. As can be observed, the optimal solution can be identified by the grid search method, though there is no closed form. On the other hand, the only upgrade strategy leads to $q_2^* = 0.33$ and $q_3^* = 0.54$, with the total expected leasing cost being \$46924.7. While the only PM strategy leads to $(n_1^*, m_1^*) = (6, 5)$, $(n_2^*, m_2^*) = (4, 4)$, and $(n_3^*, m_3^*) = (5, 4)$, with the total expected leasing cost being \$37389.7. Notice that, the total expected leasing cost would be \$48586.7, if neither upgrade nor PM actions are carried out. Therefore, in this case, it is worthwhile to perform both upgrade and PM actions to reduce the lease servicing cost, from the lessor's perspective. This is not surprising since the only PM and only upgrade strategies are special cases of the proposed upgrade and PM strategy, as shown in Section 3.3.1. In other words, their feasible domains are subsets of that of the upgrade and PM strategy, thus their optimal solutions cannot be better than that of the combined strategy.

To obtain more insights on the effect of input uncertainty on the results, comprehensive sensitivity analyses are presented in the following sections, by varying one or two parameters at a time while holding the others unchanged.

Table 1. Summary of the optimal maintenance decisions.

	Upgrade & PM			Only upgrade			Only PM		
	$j=1$	$j=2$	$j=3$	$j=1$	$j=2$	$j=3$	$j=1$	$j=2$	$j=3$
q_j^*	–	0.12	0.47	–	0.33	0.54	–	–	–
n_j^*	6	4	6	–	–	–	6	4	5
m_j^*	5	4	4	–	–	–	5	4	4
$E[C_j]$	9256.8	10548.1	16966.7	11691.7	13795.1	21437.9	9256.8	10562.2	17570.7
TC		36771.7			46924.7			37389.7	

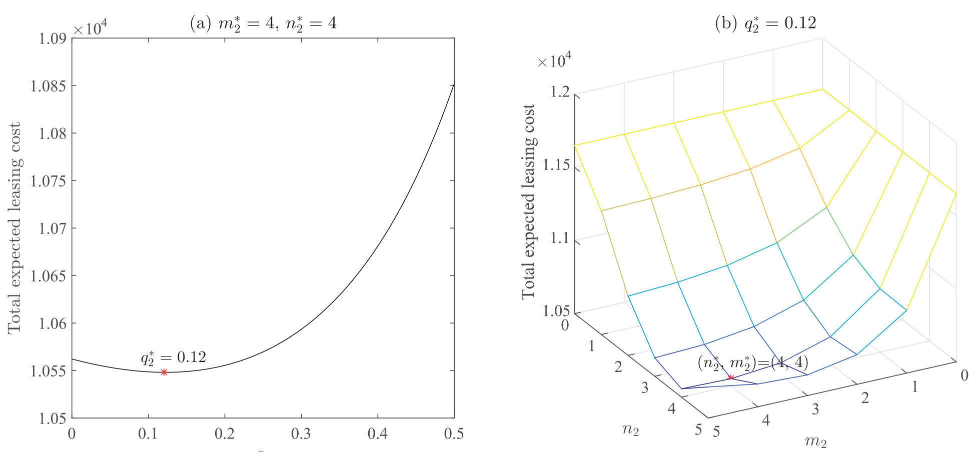

Figure 4. The existence of the optimal upgrade and PM decisions for the second lease period.

4.1.1. Sensitivity analysis of C_f and C_n

It is interesting to investigate the impacts of minimal repair cost C_f and penalty cost C_n on the results. The optimal upgrade and/or PM decisions for various combinations of C_f and C_n are summarised in Table 2. It is necessary to point out that the Type I cost C_t has similar effects on the optimal solutions to the Type II cost C_n, which can be easily known from Equations (13) and (16). Therefore, we focus only on C_n in this analysis.

From Table 2, we have the following observations:

(i) The optimal upgrade degree q_j^*, optimal number of PM actions n_j^*, optimal PM degree m_j^* and the corresponding lease servicing cost during the *j*th lease period tend to increase as the minimal repair cost C_f and/or the Type II penalty cost C_n increases. This is to be expected since the lease servicing cost tends to increase when the repair cost and/or penalty cost increases. Hence, the lessor has to enhance the upgrade and PM efforts to improve the excavator reliability.

(ii) The proposed upgrade and PM strategy always leads to the lowest total lifecycle cost among the three maintenance strategies. This demonstrates the necessity of applying both upgrade and PM programmes for lease cost reduction purpose. Also, under the given cost structures, the only PM strategy results in lower servicing costs than the only upgrade strategy.

(iii) The optimal upgrade levels under the only upgrade strategy are higher than or equal to those under the combined strategy, i.e. $q_j^o \geq q_j^*, j = 2, 3$. This can be explained by the fact that without periodical PM activities, the upgrade efforts under the only upgrade strategy should be enhanced to mitigate the equipment degradation. This is in line with our intuition.

4.1.2. Sensitivity analysis of C_s and φ

We then perform sensitivity analysis with respect to the upgrade cost parameters C_s and φ. The results are summarised in Table 3. Note that, the optimal PM decision under the only PM strategy is independent of C_s and φ. Thus, the optimal PM decision of this strategy, which has been listed in Table 1, is omitted in Table 3.

The following findings can be drawn:

(i) The optimal upgrade degree q_j^*, optimal number of PM actions n_j^*, and optimal PM degree m_j^* during the *j*th lease period tend to decrease as C_s increases, while increase as φ increases. This is reasonable since when C_s increases and/or φ decreases, the lessor would lower the upgrade degree so as to avoid heavy upgrade expenses.

(ii) The total lease servicing costs under the only upgrade strategy are higher than that under the only PM strategy (\$37389.7; see Table 1), which implies that periodical PM activities within each lease period is more cost-efficient than the one-time upgrade procedure at the beginning of the lease period. This may be due to the maintenance insufficiency in the first lease period (no upgrade and PM), which cannot prevent the equipment degradation during subsequent lease periods effectively. This finding is reflected by the current maintenance practice in the sense that industrial equipment should be subject to regular PM programmes, which include a set of maintenance tasks such as cleaning, lubricating, adjusting, and/or replacing degraded components.

4.1.3. Sensitivity analysis of the OEM warranty term

As discussed earlier, the warranty term might have an important influence on the lessor's total servicing cost. Figure 5 shows the total lifecycle lease cost under various warranty terms. It can be seen that the total lifecycle lease cost gradually decreases with the length of the OEM warranty period, regardless of the maintenance strategies applied. The implication of this finding is that the lessor should take the OEM warranty into account when making optimal maintenance decisions, since the corrective maintenance cost under warranty is borne by the OEM.

4.1.4. Sensitivity analysis of the number of lease periods

The numerical analyses above are performed based on three lease periods, i.e. $J = 3$. It is meaningful to investigate the effectiveness of the three maintenance strategies under different numbers of lease periods (i.e. J). Figure 6 shows the total lifecycle lease cost of the three maintenance strategies under different J, with $(L_1, r_1) = (36, 0.151)$, $(L_2, r_2) = (48, 0.130)$, $(L_3, r_3) = (30, 0.173)$, and $(L_4, r_4) = (48, 0.130)$. It is intuitive to observe that the total lifecycle cost increases as the number of lease period J increases. Moreover, the total lifecycle cost of the combined upgrade and PM strategy is always lower than those of the other two strategies, and the cost difference between them shows upward trend as J increases. This observation

Table 2. Comparison of the three maintenance strategies with respect to C_f and C_n.

(C_f, C_n)		Upgrade & PM			Only upgrade			Only PM		
		$j=1$	$j=2$	$j=3$	$j=1$	$j=2$	$j=3$	$j=1$	$j=2$	$j=3$
(50, 50)	q_j^*	–	0.00	0.22	–	0.00	0.31	–	–	–
	n_j^*	5	3	3	–	–	–	5	3	3
	m_j^*	4	3	4	–	–	–	4	3	3
	$E[C_j]$	5266.0	5986.7	9986.5	6277.8	7443.3	12132.0	5266.0	5986.7	10035.7
	TC		21239.2			25853.1			21288.4	
(100, 50)	q_j^*	–	0.00	0.34	–	0.06	0.43	–	–	–
	n_j^*	5	3	4	–	–	–	5	3	3
	m_j^*	4	4	4	–	–	–	4	4	4
	$E[C_j]$	6317.0	7671.4	12468.7	7729.0	9607.4	15397.5	6317.0	7671.4	12638.1
	TC		26457.1			32733.9			26626.5	
(150, 50)	q_j^*	–	0.02	0.43	–	0.23	0.49	–	–	–
	n_j^*	5	3	5	–	–	–	5	3	4
	m_j^*	5	4	4	–	–	–	5	4	4
	$E[C_j]$	7336.0	9113.4	14858.1	9180.2	11724.8	18423.0	7336.0	9113.6	15208.8
	TC		31307.5			39328.0			31658.4	
(50, 100)	q_j^*	–	0.02	0.43	–	0.23	0.49	–	–	–
	n_j^*	5	3	5	–	–	–	5	3	4
	m_j^*	5	4	4	–	–	–	5	4	4
	$E[C_j]$	8271.7	9113.4	14858.1	10240.5	11724.8	18423.0	8271.7	9113.6	15208.8
	TC		32243.2			40388.3			32594.2	
(100, 100)	q_j^*	–	0.12	0.47	–	0.33	0.54	–	–	–
	n_j^*	6	4	6	–	–	–	6	4	5
	m_j^*	5	4	4	–	–	–	5	4	4
	$E[C_j]$	9256.8	10548.1	16966.7	11691.7	13795.1	21437.9	9256.8	10562.2	17570.7
	TC		36771.7			46924.7			37389.7	
(150, 100)	q_j^*	–	0.17	0.50	–	0.40	0.58	–	–	–
	n_j^*	8	5	7	–	–	–	8	5	5
	m_j^*	5	4	4	–	–	–	5	4	4
	$E[C_j]$	10218.3	11838.6	19000.1	13142.9	15831.5	24430.0	10218.3	11871.2	19806.3
	TC		41057.0			53404.4			41895.7	
(50, 200)	q_j^*	–	0.17	0.50	–	0.40	0.58	–	–	–
	n_j^*	8	5	7	–	–	–	8	5	5
	m_j^*	5	4	4	–	–	–	5	4	4
	$E[C_j]$	11091.1	11838.6	19000.1	14203.3	15831.5	24430.0	11091.1	11871.2	19806.3
	TC		41929.8			54464.8			42768.5	
(100, 200)	q_j^*	–	0.26	0.53	–	0.46	0.61	–	–	–
	n_j^*	8	6	8	–	–	–	8	5	6
	m_j^*	5	4	4	–	–	–	5	4	4
	$E[C_j]$	12032.8	13324.1	21087.3	15654.5	17841.6	27386.9	12032.8	13400.4	22389.2
	TC		46444.2			60882.9			47822.4	
(150, 200)	q_j^*	–	0.31	0.58	–	0.50	0.63	–	–	–
	n_j^*	8	6	7	–	–	–	8	6	7
	m_j^*	5	4	5	–	–	–	5	4	4
	$E[C_j]$	12974.5	14784.0	23253.5	17105.6	19830.4	30341.3	12974.5	14927.9	24784.4
	TC		51012.0			67277.3			52686.8	

Table 3. Comparison of the three maintenance strategies with respect to C_s and φ.

		Upgrade & PM			Only upgrade		
(C_s, φ)		$j=1$	$j=2$	$j=3$	$j=1$	$j=2$	$j=3$
(1, 0.01)	q_j^*	–	0.80	0.86	–	0.86	0.87
	n_j^*	6	6	7	–	–	–
	m_j^*	5	4	5	–	–	–
	$E[C_j]$	9256.8	9577.1	14372.5	11691.7	12045.7	18391.2
	TC		33206.5			42128.7	
(5, 0.01)	q_j^*	–	0.46	0.64	–	0.60	0.69
	n_j^*	6	5	7	–	–	–
	m_j^*	5	4	4	–	–	–
	$E[C_j]$	9256.8	10323.0	15918.0	11691.7	13271.8	20179.0
	TC		35497.8			45142.5	
(10, 0.01)	q_j^*	–	0.12	0.47	–	0.33	0.54
	n_j^*	6	4	6	–	–	–
	m_j^*	5	4	4	–	–	–
	$E[C_j]$	9256.8	10548.1	16966.7	11691.7	13795.1	21437.9
	TC		36771.7			46924.7	
(1, 0.05)	q_j^*	–	0.92	0.94	–	0.94	0.95
	n_j^*	6	7	8	–	–	–
	m_j^*	5	4	5	–	–	–
	$E[C_j]$	9256.8	9063.1	13575.7	11691.7	11304.6	17528.1
	TC		31895.6			40524.5	
(5, 0.05)	q_j^*	–	0.80	0.86	–	0.86	0.87
	n_j^*	6	6	7	–	–	–
	m_j^*	5	4	5	–	–	–
	$E[C_j]$	9256.8	9593.5	14381.8	11691.7	12150.4	18433.7
	TC		33232.1			42275.9	
(10, 0.05)	q_j^*	–	0.69	0.80	–	0.78	0.81
	n_j^*	6	5	7	–	–	–
	m_j^*	5	4	5	–	–	–
	$E[C_j]$	9256.8	9918.7	15005.1	11691.7	12722.6	19100.7
	TC		34180.6			43515.1	
(1, 0.10)	q_j^*	–	0.95	0.96	–	0.96	0.96
	n_j^*	6	7	8	–	–	–
	m_j^*	5	4	5	–	–	–
	$E[C_j]$	9256.8	8919.8	13386.7	11691.7	11100.3	17308.3
	TC		31563.3			40100.3	
(5, 0.10)	q_j^*	–	0.87	0.90	–	0.90	0.91
	n_j^*	6	6	8	–	–	–
	m_j^*	5	4	5	–	–	–
	$E[C_j]$	9256.8	9338.2	13982.3	11691.7	11787.5	17989.9
	TC		32577.3			41469.1	
(10, 0.10)	q_j^*	–	0.80	0.86	–	0.86	0.87
	n_j^*	6	6	7	–	–	–
	m_j^*	5	4	5	–	–	–
	$E[C_j]$	9256.8	9614.5	14393.6	11691.7	12295.4	18489.3
	TC		33264.9			42476.4	

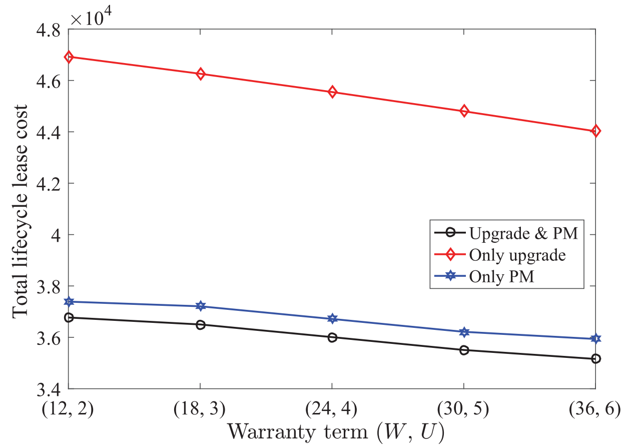

Figure 5. Impact of the warranty term on the total lifecycle lease servicing cost.

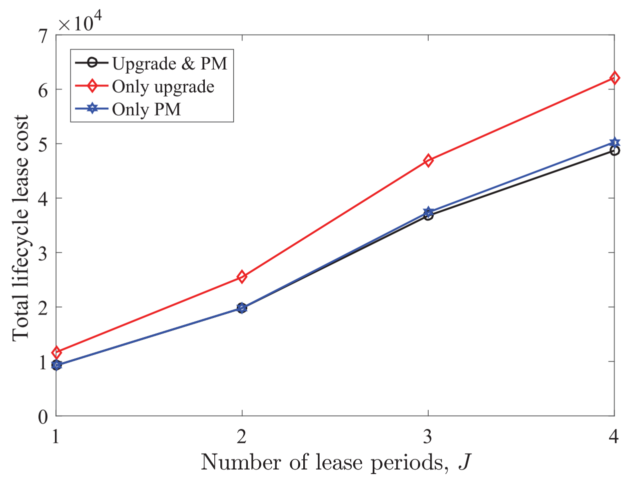

Figure 6. Impact of the number of lease periods, J, on the total lifecycle lease servicing cost.

is essential for lessors because the cost benefit of implementing upgrade and PM activities becomes larger when a piece of equipment goes through more lease contracts (and its total lease duration becomes longer).

4.2. Additional numerical experiments

In this section, additional numerical experiments are conducted, based on different and randomly-generated model parameters. This will help us further validate the proposed maintenance model and strategy, and gain more managerial insights.

In this experiment, we focus on three sets of model parameters, i.e. Weibull parameters (α and β), minimal repair and penalty costs (C_f and $C_n + \hat{C}_t$), and upgrade cost parameters (C_s and φ), which are key parameters associated with the maintenance optimisation problem. Here, we select three levels for each parameter set. In total, there are 27 combinations of these key parameters; see Table 4 for details. Moreover, the parameter settings of PM level $\delta(m_j)$ and PM cost $C_p(m_j)$ are the same as those in Section 4.1. Other parameters are arbitrarily set as $\gamma = 1$, $r_0 = 1$, $W = 1$, $U = 2$, $(L_1, r_1) = (3, 0.8)$, $(L_2, r_2) = (4, 1.2)$, and $(L_3, r_3) = (2, 0.8)$. In this experiment, the units for time and usage are changed to year and 10^4 kilometres, respectively.

The optimal lifecycle leasing costs of the proposed upgrade and PM strategy under different parameter combinations are shown in Figure 7, along with those of the only PM and only upgrade strategies. The following findings can be obtained:

(i) The total expected lifecycle leasing costs of the upgrade and PM strategy are always lower than those of the only PM and only upgrade strategies. This further demonstrates the cost effectiveness of this combined strategy, as discussed earlier.

Table 4. Combinations of the model parameters.

Index	α	β	C_f	$C_n + \hat{C}_t$	C_s	φ
1	1.5	1.5	85	120	6.5	0.04
2					3.5	0.04
3					3.5	0.08
4			85	150	6.5	0.04
5					3.5	0.04
6					3.5	0.08
7			150	150	6.5	0.04
8					3.5	0.04
9					3.5	0.08
10	1.5	2.5	85	120	6.5	0.04
11					3.5	0.04
12					3.5	0.08
13			85	150	6.5	0.04
14					3.5	0.04
15					3.5	0.08
16			150	150	6.5	0.04
17					3.5	0.04
18					3.5	0.08
19	1	2.5	85	120	6.5	0.04
20					3.5	0.04
21					3.5	0.08
22			85	150	6.5	0.04
23					3.5	0.04
24					3.5	0.08
25			150	150	6.5	0.04
26					3.5	0.04
27					3.5	0.08

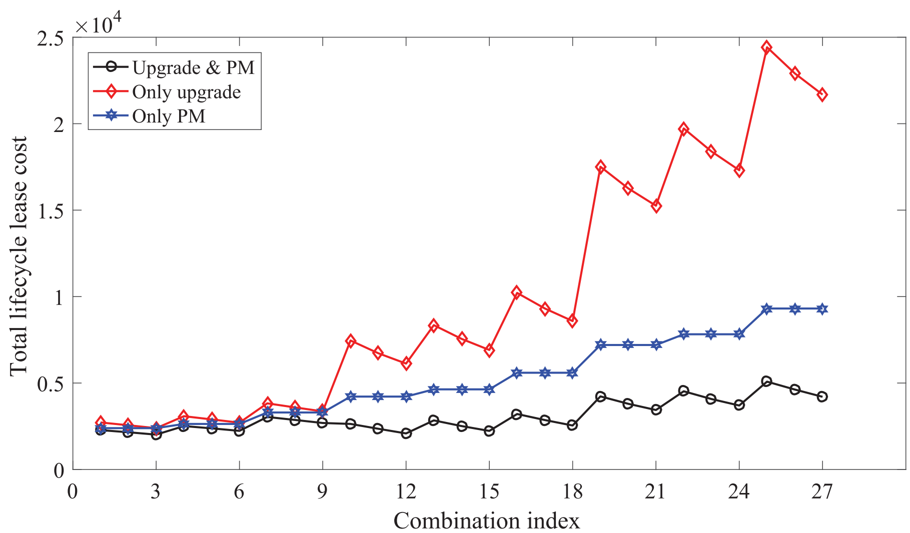

Figure 7. Total lifecycle lease servicing costs for different combinations of model parameters.

(ii) It can be observed that under parameter combination #3, the lifecycle leasing cost of the only upgrade strategy is even lower than that of the only PM strategy. This is because that under this combination, the equipment failure probability is relatively low, the repair and penalty costs are small, and the upgrade cost is small (with small C_s and large φ). In this case, pre-leasing upgrade action at the beginning of each lease period is effective enough for reducing the total leasing expenses. Nevertheless, this scenario may be rare in real applications.

(iii) Moreover, when the Weibull scale parameter α becomes smaller and/or the shape parameter β becomes larger (corresponding to higher equipment failure rate), the lease cost growth under the upgrade and PM strategy is much less significant than those under the only upgrade and only PM strategies, as shown in Figure 7. This also indicates the superiority of this combined strategy.

(iv) Furthermore, Figure 7 shows that the lifecycle lease servicing cost increases as C_f and/or $C_n + \hat{C}_t$ increases; while it decreases as C_s decreases and/or φ increases. These observations are consistent with those before.

5. Concluding remarks

In this work, for the first time, the OEM warranty, the usage-based lease contract and the successive leasing manner are integrated into the maintenance optimisation problem of leased industrial equipment. A modelling and optimisation framework of a progressive upgrade and PM updating strategy is proposed to assist equipment lessors in making optimal decisions on upgrade and PM activities during successive lease periods. Since industrial equipment generally operates under different usage rates during different lease periods, a new PM modelling framework is developed by combining the AFT model, the age reduction model with the concept of statistical virtual age. Essentially, the progressive nature of the proposed maintenance optimisation problem due to the successive leasing manner is highlighted.

Overall, this paper, on the one hand, provides a quantitative maintenance modelling and optimisation framework and, on the other hand, draws several managerial insights/findings from the numerical studies, to facilitate lessors' maintenance decision-making process. The main finding is that it is worthwhile to implement both pre-leasing upgrade and post-leasing PM strategies to minimise the total lease servicing cost. Nevertheless, under the given cost structures, the periodical PM actions within each lease period appear to be more effective than the one-time upgrade procedure at the beginning of the lease period, especially when the equipment failure rate is high. Consequently, the lessors should focus more on regular maintenance tasks such as cleaning, lubricating, adjusting, and/or replacing degraded components, within each lease period. Furthermore, the cost benefit of applying upgrade and PM activities becomes larger, when the equipment failure rate is higher, the equipment goes through more lease contracts, and/or the OEM warranty is incorporated. These findings and insights would be of importance to equipment lessors who seek the minimum cost to be generated from their leased industrial equipment.

This study can be extended in several directions.

- In this paper, we considered that periodical PM actions are performed within each lease period; while aperiodic PM policy with decreasing PM interval is of potential interest to further reduce the leasing cost.
- Besides, it is more realistic to consider that PM actions have non-negligible durations.
- Relaxing the minimal repair assumption and considering imperfect repairs also have practical value.
- Finally, this paper made the assumption that no information regarding the future usage rates will be known in advance. However, it may be more interesting to consider the future usage in certain ways, e.g. the expected usage, if the lessor manages a large fleet of industrial equipment for leasing purpose. This will help the lessor take into account the whole equipment lifecycle, instead of focusing solely on each single lease period, when making optimal maintenance decisions.

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Appendix

In Section 3, the optimisation model is derived in the case where the actual warranty length is shorter than the first lease period, i.e. $\min\{W, U/r_1\} \le L_1$. In this appendix, we present the optimisation model in a general case where the warranty period terminates within the hth lease period, i.e. $\sum_{i=1}^{h-1} L_i < W < \sum_{i=1}^{h} L_i$ and $\sum_{i=1}^{h-1} L_i r_i < U < \sum_{i=1}^{h} L_i r_i$, $h = 1, 2, \ldots, J$; see Figure A1. In this case, h can be obtained by

$$h = \min\left\{ j \,\middle|\, \sum_{i=1}^{j} L_i > W \text{ or } \sum_{i=1}^{j} L_i r_i > U \right\}.$$

As a result, the actual warranty length W_r in the time dimension should be

$$W_r = \begin{cases} W, & \text{if } r_h \le \bar{r}_h, \\ \sum_{i=1}^{h-1} L_i + \left(U - \sum_{i=1}^{h-1} L_i r_i \right) \Big/ r_h, & \text{if } r_h > \bar{r}_h, \end{cases}$$

where $\bar{r}_h = (U - \sum_{i=1}^{h-1} L_i r_i)/(W - \sum_{i=1}^{h-1} L_i)$.

It is worth mentioning that h and W_r can be known only after the $(h-1)$th lease period. That is to say, we cannot know h and W_r in advance, e.g. at the very beginning of the equipment lifecycle. This is also due to the successive leasing manner, which is the same as the scenario of determining J.

When the warranty period terminates within the hth lease period, the number of PM actions implemented within the warranty period $[0, W_r]$ is

$$\sum_{i=1}^{h-1} n_i + d = \sum_{i=1}^{h-1} n_i + \max\left\{ k \,\middle|\, \sum_{i=1}^{h-1} L_i + k\Delta_h < W_r \right\}. \tag{A1}$$

Then, the expected number of equipment failures within the period $(W_r, \sum_{i=1}^{h} L_i]$ is given by

$$E\left[N_h^{\Phi}(q_h, n_h, m_h \mid \mathcal{H}_h)\right] = \int_{v_{h,d}+W_r-\sum_{i=1}^{h-1} L_i - d\Delta_h}^{v_{h,d}+\Delta_h} \lambda(t \mid r_h)\,\mathrm{d}t + \sum_{k=d+1}^{n_h} \int_{v_{h,k}}^{v_{h,k}+\Delta_h} \lambda(t \mid r_h)\,\mathrm{d}t, \tag{A2}$$

where d can be obtained from Equation (A1).

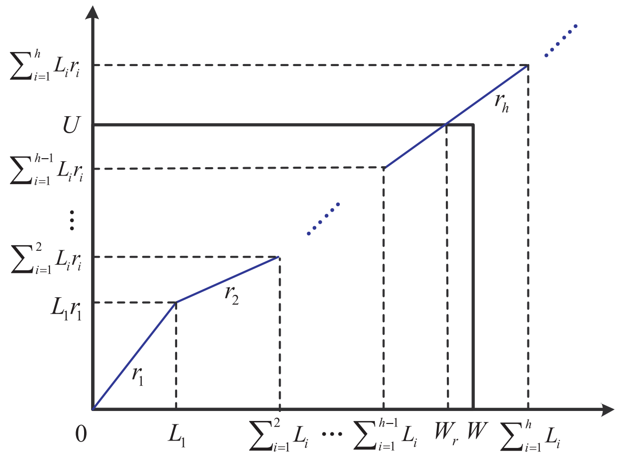

Figure A1. Illustration of the expiry of the warranty period in a general case.

Therefore, when the OEM warranty terminates within the hth lease period, the lessor's total expected servicing cost within the jth, $j = 1, 2, \ldots, J$, lease period is given by

$$E\left[C_j\left(q_j, n_j, m_j \mid \mathcal{H}_j\right)\right] = \begin{cases} \left(\hat{C}_t + C_n\right) E\left[N_1(n_1, m_1)\right] + n_1 C_p(m_1), & j = 1, \\ \left(\hat{C}_t + C_n\right) E\left[N_j\left(q_j, n_j, m_j \mid \mathcal{H}_j\right)\right] + C_u(q_j) + n_j C_p(m_j), & 2 \le j \le h-1, \\ \left(\hat{C}_t + C_n\right) E\left[N_j\left(q_j, n_j, m_j \mid \mathcal{H}_j\right)\right] + C_u(q_j) & \\ \quad + n_j C_p(m_j) + C_f E\left[N_j^{\Phi}\left(q_j, n_j, m_j \mid \mathcal{H}_j\right)\right], & j = h, \\ \left(\hat{C}_t + C_n + C_f\right) E\left[N_j\left(q_j, n_j, m_j \mid \mathcal{H}_j\right)\right] + C_u(q_j) + n_j C_p(m_j), & h+1 \le j \le J, \end{cases} \tag{A3}$$

where $C_u(q_j)$ is given by (7), $E[N_1(n_1, m_1)]$ by (12), $E[N_j(q_j, n_j, m_j \mid \mathcal{H}_j)]$, $j = 2, \ldots, J$, by (15), and $E[N_h^{\Phi}(q_h, n_h, m_h \mid \mathcal{H}_h)]$ by (A2).

The interpretation of (A3) goes like:

(i) when $j \le h-1$, the jth lease period is totally covered by the warranty period, then the lessor's servicing cost includes only the upgrade cost (except for the first period), PM cost and failure penalty costs;
(ii) when $j = h$, this lease period is partially overlapped with the warranty period, then the lessor has to pay for the minimal repair cost within $(W_r, \sum_{i=1}^{h} L_i]$, in addition to the upgrade cost, PM cost and failure penalty costs; and
(iii) when $h+1 \le j \le J$, all of the upgrade cost, PM cost, minimal repair cost, and failure penalty costs within the jth lease period are borne by the lessor.

Finally, the optimisation problem for the jth lease period is also given by (17), by replacing $E[C_j(q_j, n_j, m_j \mid \mathcal{H}_j)]$ with (A3).